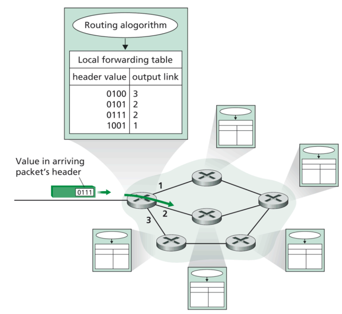
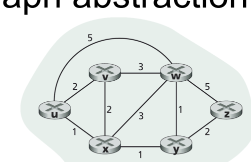
Routing Algorithms

Placing routing into context



Graph abstraction



Graph: G = (N,E)

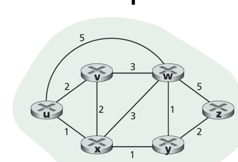
N = set of routers = { u, v, w, x, y, z }

E = set of links ={ (u,v), (u,x), (v,x), (v,w), (x,w), (x,y), (w,y), (w,z), (y,z) }

Remark: Graph abstraction is useful in other network contexts

Example: P2P, where N is set of peers and E is set of TCP connections

Graph abstraction: costs



- c(x,x') = cost of link (x,x')
  - e.g., c(w,z) = 5
- cost could always be 1, or inversely related to bandwidth, or inversely related to congestion

Cost of path $(x_1, x_2, x_3,\ldots, x_p) = c(x_1,x_2) + c(x_2,x_3) + \ldots + c(x_{p-1},x_p)$

What's the least-cost path between u and z ?

Routing algorithm: algorithm that finds least-cost path

Routing Algorithm classification

Global or decentralized information?

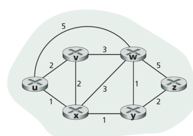
Global:
- all routers have complete topology, link cost info
- "link state" algorithms

Decentralized:
- router knows physically-connected neighbors, link costs to neighbors
- iterative process of computation, exchange of info with neighbors
- "distance vector" algorithms

Static or dynamic?

Static:
- routes change slowly over time

Dynamic:
- routes change more quickly
  - periodic update
  - in response to link cost changes



A Link-State Routing Algorithm

Dijkstra's algorithm
- net topology, link costs known to all nodes
  - accomplished via "link state broadcast"
  - all nodes have same info
- computes least cost paths from one node ('source") to all other nodes
  - gives forwarding table for that node
- iterative: after k iterations, know least cost path to k dest.'s

Notation:
- c(x,y): link cost from node x to y; = ∞ if not direct neighbors
- D(v): current value of cost of path from source to dest. v
- p(v): predecessor node along path from source to v
- N': set of nodes whose least cost path definitively known

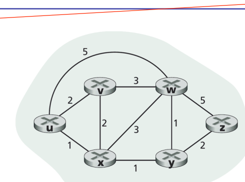
Dijsktra's Algorithm

```
Initialization:
  N' = {u}
  for all nodes v
    if v adjacent to u
        then D(v) = c(u,v)
    else D(v) = ∞

Loop
  find w not in N' such that D(w) is a minimum
   add w to N'
   update D(v) for all v adjacent to w and not in N' :
      D(v) = min( D(v), D(w) + c(w,v) )
   /* new cost to v is either old cost to v or known
    shortest path cost to w plus cost from w to v */
until all nodes in N'
```

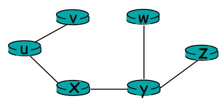
Dijkstra's algorithm: example

Step	N'	D(v),p(v)	D(w),p(w)	D(x),p(x)	D(y),p(y)	D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2	uxy	2,u	3,y			4,y
3	uxyv		3,y			4,y
4	uxyvw					4,y
5	uxyvwz					



Dijkstra's algorithm: an example

Resulting shortest-path tree from u:



Resulting forwarding table in u:

destination	link
v	(u,v)
x	(u,x)
y	(u,x)
w	(u,x)
z	(u,x)

Distance Vector Algorithm

Bellman-Ford Equation

Define

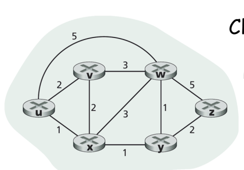
$d_x(y)$:= cost of least-cost path from x to y

Then

$$\boxed{d_x(y) = \min_v \{c(x,v) + d_v(y)\}}$$

where min is taken over all neighbors v of x

Bellman-Ford example



Clearly, $d_v(z) = 5$, $d_x(z) = 3$, $d_w(z) = 3$

B-F equation says:

$$\begin{aligned} d_u(z) &= \min \{ c(u,v) + d_v(z), \\ &\qquad c(u,x) + d_x(z), \\ &\qquad c(u,w) + d_w(z) \} \\ &= \min \{2 + 5, \\ &\qquad 1 + 3, \\ &\qquad 5 + 3\} = 4 \end{aligned}$$

Node that achieves minimum is next hop in shortest path ➜ forwarding table

Distance Vector Algorithm

- $D_x(y)$ = estimate of least cost from x to y
- Node x knows cost to each neighbor v: c(x,v)
- Node x maintains distance vector $\mathbf{D}_x = [D_x(y): y \in N]$
- Node x also maintains its neighbors' distance vectors
  - For each neighbor v, x maintains $\mathbf{D}_v = [D_v(y): y \in N]$

Distance vector algorithm

- Each node periodically sends its own distance vector estimate to neighbors
- When a node x receives new DV estimate from neighbor, it updates its own DV using B-F equation:

$D_x(y) \leftarrow \min_v\{c(x,v) + D_v(y)\}$ *for each node* $y \in N$

- Estimate $D_x(y)$ converge to the actual least cost $d_x(y)$

Distance Vector Algorithm

Iterative, asynchronous:
each local iteration caused by:
- local link cost change
- DV update message from neighbor

Distributed:
- each node notifies neighbors *only* when its DV changes
  - neighbors then notify their neighbors if necessary

Each node:

wait for (change in local link cost or msg from neighbor)

recompute estimates

if DV to any dest has changed, *notify* neighbors

$D_x(y) = \min\{c(x,y) + D_y(y), c(x,z) + D_z(y)\}$
$= \min\{2+0 , 7+1\} = 2$

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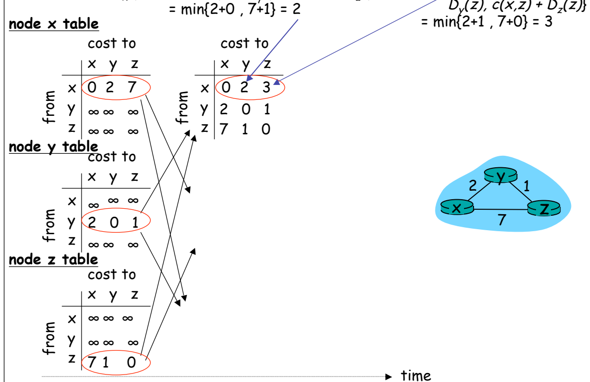

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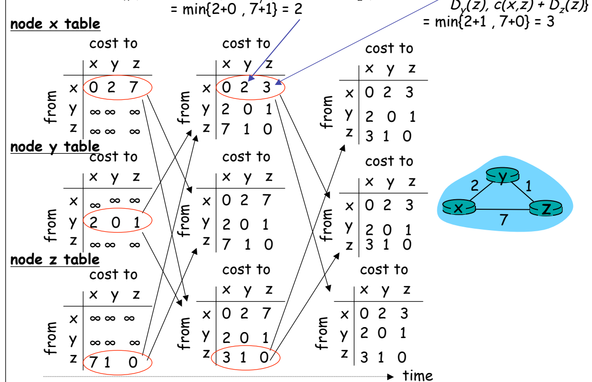
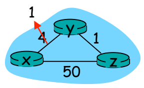


Distance Vector: link cost changes

Link cost changes:

- node detects local link cost change
- updates routing info, recalculates distance vector
- if DV changes, notify neighbors



"good news travels fast"

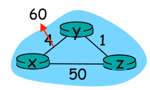
At time t_0, *y* detects the link-cost change, updates its DV, and informs its neighbors.

At time t_1, *z* receives the update from *y* and updates its table. It computes a new least cost to *x* and sends its neighbors its DV.

At time t_2, *y* receives *z*'s update and updates its distance table. *y*'s least costs do not change and hence *y* does *not* send any message to *z*.

Distance Vector: link cost changes

Link cost changes:



- good news travels fast
- bad news travels slow - "count to infinity" problem!
- 44 iterations before algorithm stabilizes: see text
- Poisoned reverse:
- If Z routes through Y to get to X :
  - Z tells Y its (Z's) distance to X is infinite (so Y won't route to X via Z)
- will this completely solve count to infinity problem?