A Simulation of Auroral Absorption


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by

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Abstract

HF radio transmissions propagate long distances by reflecting off the ionosphere. At high latitudes radio propagation is strongly affected by the northern lights (aurora borealis), which causes ionization at low altitudes and hence the absorption of radio waves. Models of this process are still in a primitive state. A simulation of radio wave propagation was created in order to test Foppiano and Bradley’s empirical model of auroral absorption. The simulation attempts to predict the net absorption of signals at a receiver by simulating a large number of transmitters, even though the exact sources of the signals are unknown. Although the simulation takes into account auroral and nonauroral absorption as well as other sources of path loss, the analysis focuses on the nighttime aurora. An intelligent search algorithm is used in order to efficiently adjust the model to best fit the data. The output of the simulation is qualitatively and quantitatively compared to signal levels observed with HF radio receivers located in northern Canada. The analysis allows us to develop alternative models of auroral absorption which account for the level of geomagnetic activity, and these are compared to the standard Foppiano and Bradley model.
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Chapter 1

Introduction and Background

1.1 Overview of Auroral Absorption

The aurora and its spectacular displays of color in the night sky has for a long time been the subject of fascination and scientific research. Besides its visual beauty, the aurora has a profound effect on radio wave propagation, since auroral absorption can significantly reduce the amount of signal strength received. It is very important to account for auroral absorption when planning how much power is required to transmit across polar regions. Also studying auroral absorption can help predict radio blackout occurrences in these regions. The aurora therefore greatly affects radio communications and modeling auroral absorption is thus very important to radio operators and the telecommunications industry.

Foppiano and Bradley [1983] proposed a model to predict the auroral absorption of high frequency radio waves at vertical incidence which can be related to oblique incidence. In this thesis we will apply the Foppiano and Bradley model [1983], in an attempt to simulate the auroral absorption of 3.6 - 4.5 MHz radio waves propagating long distances from temperate latitude transmitters to polar latitude receivers. The
results of the simulation will then be qualitatively and quantitatively compared to data taken from radio receivers stationed in northern Canada in order to experimentally test the Fopiano and Bradley model. We will attempt to improve upon the model by trying to fit some of its variables to our data. The results will be used to help propose possible modifications to the Fopiano and Bradley model and our
simulation based on it.

Auroral absorption is caused by the interaction of particles from space with the earth’s ionosphere, creating ionization in the D region of the ionosphere. Foppiano and Bradley model these types of particle precipitation into two components: splash, which occurs around midnight, and drizzle, which occurs during the morning hours. The splash term corresponds to the substorm aurora while the drizzle corresponds to the diffuse aurora. Foppiano and Bradley present an empirical model to describe the effects of both these families of precipitating particles on high frequency radio waves, which is based upon geomagnetic location, magnetic local time, solar epoch, and time of year. They have also presented an alternate model to take into account the geomagnetic activity by introducing the Kp index into the latitude dependence of both the splash and drizzle terms. Part of the data analysis will involve studying the differences between the Kp dependent and the non-Kp dependent auroral absorption models in an attempt to determine which model better reflects the data.

1.2 Previous Studies of Auroral Absorption

Foppiano and Bradley [1983] created an empirical model of auroral absorption based on large quantities of riometer data. The riometer is a simple instrument that measures the level of cosmic radio noise. It works on the principle that the amount of cosmic radiation is relatively constant, and therefore any drops in the measured level can be attributed to ionospheric absorption. In order to detect drops due to auroral absorption, riometer data is generally compared to a ‘quiet day’ curve where the amount of auroral absorption in minimal. From their data they developed a model to predict $Q_1$, the percentage of days where the absorption measured by a riometer at 30 MHz exceeds 1 dB. Their model for $Q_1$ can be converted to absolute absorption
(in dB) and applied to other HF frequencies using equations provided in Foppiano and Bradley, [1983]. It is this model which we will explore in detail, test using using our data from Canadian receivers, and modify to better describe our data.

Milan et al. [1995] conducted a test of the Foppiano and Bradley model using a 6.8 MHz signal propagating along a 3200 km path across the auroral oval. Unlike the Foppiano and Bradley study, which examined cosmic noise levels, this test studied radio signals propagating from a known transmitter. The advantage of this test is that it directly studies the auroral absorption effects of man-made signals, which are directly applicable to telecommunications. Unlike cosmic waves which are received at vertical incidence, man-made transmissions propagating along a long path are reflected off the ionosphere and are thus received at oblique incidence. However, these signals can be analyzed using the Foppiano and Bradley system for converting vertical incidence absorption to oblique incidence absorption. Milan found that his data tended to agree with the Foppiano and Bradley model. In addition he found that his data tended to exhibit a strong geomagnetic dependence when data for different values of Kp, a geomagnetic activity index were analyzed.

1.3 A New Test of the Foppiano and Bradley Model

This thesis we will explore a different means of testing the Foppiano and Bradley model, by using a variation of the Milan test. It is often logistically difficult to operate radio transmitters, but it is important to study the auroral effects on man-made transmissions. Therefore we have developed a test that utilizes only a HF receiver and studies man-made transmissions from already previously working transmitters. One of the difficulties of this test is that the exact locations of the signal sources are not known. Furthermore received signals may have been propagating along several
different paths. To resolve this a computer simulation was developed which take as input the locations of several possible sources for these signals, or 'virtual transmitters', and computes the possible paths for the signals propagating from each virtual transmitter. Using Foppiano and Bradley’s model for auroral absorption at oblique incidence, and models in Davies [1965] for photoionization, spatial attenuation, and ground loss, the simulation calculates the total loss along each path. All the path losses are combined into a net loss for the receiver using the inverse sum equation also provided by Davies [1965]. This test relies on the assumption that the geometry of virtual transmitters used is a reasonable approximation to the actual locations of the sources. For this thesis the geometry of virtual transmitters will be an even grid over the United States and southern Canada, though there are other reasonable geometries such as locating the virtual transmitters near major cities.

Data collected from HF radio receivers located in northern Canada are used to test the Foppiano and Bradley model, as well as the viability of the simulation as a new test of this model. The receivers are nearly located along the same longitude, which enables us to study the latitude dependences of the aurora in the model. Furthermore data were taken over the course of several years, with different sunspot numbers and different levels of geomagnetic activity which allows us to explore the impacts of both of these factors on the model. Since the Foppiano and Bradley model is based on statistical results, a large number of days for each site and year were incorporated into the study, and the average signal level was used for comparison to the model.

Lastly, we will quantitatively compare the simulation's results to the data taken by the radio receivers and attempt to fit the model to the data. Since there are potentially several variables in the Foppiano and Bradley model that can be fit to data, the search space is quite large. It is therefore not feasible to test every possible combination of values for the variables to be fitted. Thus an efficient hill-climbing
Figure 1.2: The overall design of the software. Ovals represent input/output and the rectangles represent software execution.

search was implemented to search for a good fit. For the algorithm to work correctly it is important that a good heuristic be selected. The standard error as calculated using the principle of least squares fitting is used as the heuristic since it quantitatively measures the goodness of fit between the model curve and the data.

A modeling and analysis software system was developed in order to perform this new test of the Foppiano and Bradley model. Figure 1.2 shows the software design, which consists of three branches consisting of the simulation, a data processing module, and a search module to fit the model to our data. The simulation, which implements the Foppiano and Bradley model, is discussed in more detail in Chapters 2 and 3. Chapter 4 discusses how the software system handles and manipulates data taken from HF radio receivers located in northern Canada, and Chapter 5 presents the implementation and performance analysis of the search module. Also a user refer-
ence guide to the software can be found in Appendix C. The three branched software
design provides an efficient and effective means of comparing the model results to
data and for adjusting the model to better fit the data.

This thesis explores the ability of the Foppiano and Bradley model to describe
data taken from receivers at auroral latitudes as well as how factors influencing the
Kp index affect the auroral absorption. It also explores the ability for simulations
and search algorithms to test and model physical phenomena. Although auroral
absorption will be the focus of this thesis, the computational techniques used can
potentially be applied to studying other models with both scientific and non-scientific
applications.
Chapter 2

Predicting the Path Loss of HF Radio Waves

2.1 Introduction

The receivers stationed in Northern Canada receive HF radio broadcast signals from transmitters located throughout the United States and southern Canada. The signals propagate along many different paths, and incur path losses due to the distance of propagation, ground reflections, daytime ionospheric absorption, and auroral ionospheric absorption. The last two effects mainly occur when the signal propagates though the thin D region of the ionosphere. A computer simulation was developed using the model outlined in this chapter to estimate the overall path loss in decibels of a particular propagation path as a function of universal time, location, sunspot number, geomagnetic activity, and time of year. This calculation is then applied to a system, taking into account multiple propagation paths and multiple transmitters emitting HF radio waves throughout North America. The results will later be compared to the data of signal strengths received from the Canadian receivers.
2.2 The Absorption Model

2.2.1 Auroral Absorption at Vertical Incidence

Auroral absorption at vertical incidence can be monitored by a variety of techniques. One of the simplest is the riometer, or relative ionospheric opacity meter. A riometer monitors the ground level amplitude of background radiation of cosmic origin, typically at a frequency of 30 MHz. Cosmic radiation tends to have an incidence angle of 0 degrees and therefore is said to be received at vertical incidence. Since the source is extremely constant, variations in riometer readings are attributed to ionospheric absorption.

Foppiano and Bradley [1983] used a large database of riometer data to develop an empirical model of auroral absorption. Auroral absorption occurs in the D region of the ionosphere, at an altitude of about 85 km, and is a result of the significant number of collisions between electrons and neutrals in this region. Auroral absorption is therefore enhanced by a greater electron density in the D region, which can occur as a result of electron impact ionization during auroral events. The Foppiano and Bradley model, later referred to as FB, groups auroral absorption into two components, a drizzle component which peaks near 68 degrees in corrected geomagnetic latitude during the morning hours, and a splash component, which peaks near 67 degrees in corrected geomagnetic latitude, and close to local midnight. The drizzle and splash components of the model represent two different types of particle precipitation in the ionosphere with energies of approximately 40 and 80 keV respectively. Physically, the drizzle component corresponds to the diffuse aurora, while the splash component models substorm aurora. Since this thesis deals only with nighttime data, the drizzle component is relatively unimportant, though the drizzle terms are included for completeness. We will therefore focus our investigation on the splash components of the
Figure 2.1: Plot of $Q_1$ for a signal propagating from a simulated transmitter positioned at (38,266) to Churchill (58.5,266) using the standard Foppiano and Bradley model.

The independent variables of the FB model are geomagnetic latitude, $\lambda$, geomagnetic local time, $t$, the smoothed Zurich sunspot number, $R$, geomagnetic longitude, $\theta$, and season, $M$. From Foppiano and Bradley [1983], the principle equation of the FB model is

$$Q_1 = d_{\text{amp}} d_{\lambda} d_{R} d_{\theta} d_{m} + s_{\text{amp}} s_{\lambda} s_{R} s_{\theta} s_{m}$$

(2.1)

where $Q_1$ is the percentage of days that a 30 MHz signal at vertical incidence suffers at least a 1 dB loss due to auroral absorption.

The first term on the right side of the equation expresses the drizzle component
and the second term expresses the splash component. In the FB Model coefficients $d_{amp} = 21$ and $s_{amp} = 12$. Foppiano and Bradley give a relationship between $Q_1$ and the auroral absorption at vertical incidence expressed in power dBs, $A_m$:

$$A_m = 0.02Q_1$$  \hspace{1cm} (2.2)

$Q_1$ and $A_m$ are based upon signals propagating at 30 MHz. Foppiano and Bradley do give a relation between $A_m$ at 30 MHz and absorption at other frequencies which is further explored below in the section pertaining to oblique incidence.

### 2.2.2 The Splash Component

Of the five different variables of the splash component of the FB model, the latitude dependence is one of the most critical. In the FB model all coordinates are corrected geomagnetic, CGM, which are based upon the local magnetic field and are considered the most appropriate for modeling the aurora. The simulation converts between geographical and CGM coordinates using an algorithm provided courtesy of Milan which is based on the international geomagnetic reference field model. The latitude dependence, $s_\lambda$ is modeled using a Gaussian function as follows:

$$s_\lambda = \exp \left[ -\frac{(\lambda - \lambda'_m)^2}{2\sigma_\lambda^2} \right]$$  \hspace{1cm} (2.3)

where

$$\lambda'_m = 67(1 - 0.0006R) + 0.3(1 + 0.012R)|t|$$  \hspace{1cm} (2.4)

and

$$\sigma_\lambda = 3(1 + 0.004R)$$  \hspace{1cm} (2.5)
where $\lambda$ is the latitude in CGM coordinates and $t$ is given in terms of the geomagnetic local time $T$ as follows

$$
t = \begin{cases} 
T - 3 & \text{for } 0 \leq T \leq 15 \\
T - 27 & \text{for } 15 < T < 24
\end{cases}
$$

The peak of the Gaussian, $\lambda_m'$, is dependent upon the sunspot number $R$, and peaks around 68 degrees during a solar minimum, though during a solar maximum the splash peaks at lower latitudes. The width of the Gaussian, $\sigma_\lambda$, is about 3 degrees during a solar maximum and increases with higher values of $R$.

The diurnal component is also modeled as a Gaussian and is a function of $t_2$:

$$
s_t = \exp\left[-\frac{t_2^2}{15.7}\right]
$$

(2.6)

where

$$
t_2 = \begin{cases} 
T & \text{for } 0 \leq T \leq 12 \\
T - 24 & \text{for } 12 < T < 24
\end{cases}
$$

The conversion from universal time, UT, to magnetic local time, $T$, is calculated as follows (Milan 1995):

$$
T = \frac{\theta + 15(UT - 4.73)}{15}
$$

(2.7)

In the FB model the splash peaks around local geomagnetic midnight and has a Gaussian width of slightly less than 3 hours.

The splash term is modeled to be stronger during solar maxima than in solar minima as follows:

$$
s_r = 1 + 0.009R
$$

(2.8)
In the FB model the smoothed sunspot number is used as the value for R.

Although the latitude dependence is more important, there is also a longitudinal dependence. The FB model calculates the longitudinal dependence of the splash for the northern hemisphere as follows:

\[
s_\theta = \begin{cases} 
0.58 - 0.42 \sin[0.947(\theta + 85^\circ)] & \text{for } 0 \leq \theta < 10 \\
0.16 & \text{for } 10 \leq \theta < 80 \\
0.58 + 0.42 \sin[1.80(\theta + 130^\circ)] & \text{for } 80 \leq \theta < 130 \\
0.58 - 0.42 \sin[0.947(\theta - 275^\circ)] & \text{for } 180 \leq \theta < 360 
\end{cases}
\]

where \( \theta \) is the corrected geomagnetic longitude in degrees.

The FB model does not include a seasonal dependence for the splash precipitation and thus:

\[ s_m = 1 \quad (2.10) \]

Our data set described in Chapter 4 pertains only to one season and hence we will not address seasonal effects in this thesis. The drizzle terms of the model are similar to the splash terms and have been placed in Appendix A.

### 2.2.3 Including Kp

The level of geomagnetic activity in the ionosphere has a profound effect on both the location and intensity of the aurora and hence also affects the amount of auroral absorption. High levels of geomagnetic activity are connected with a greater auroral intensity and a more southerly auroral oval. There are several indices that are used to quantify the amount of geomagnetic activity. One of the most useful is the planetary K index, which is more commonly known as Kp. The K index is based upon the
magnitude of the range of the horizontal magnetic field component observed during each three hour interval, based on universal time. The Kp index is defined as the average of the K indices reported by each of the standard observatories.

Foppiano and Bradley also proposed a short term prediction model, which took the Kp index into account. The two models differ in the representations of the latitude dependent and the geomagnetic local time dependent terms. Since Kp indexes magnetic activity, it was shown to be a very good predictor of the peak auroral latitude for short term predictions. It is possible that Kp may be a better overall predictor of auroral latitude dependence than R as even though higher values of Kp tend to occur during periods of higher sunspot numbers, this is not always the case. The width of the latitude Gaussian of the splash precipitation, $\sigma_\lambda$, in the Kp model is given by:

$$\sigma_\lambda = 2.16(1 + 0.17Kp)$$  \hspace{1cm} (2.11)

Combining Equation 2.11 with the solar epoch based Equation 2.5 for $\sigma_\lambda$ yields the relation:

$$R = 30.6Kp - 70.0$$  \hspace{1cm} (2.12)

Although Foppiano and Bradley do not give a formula based on Kp for the peak of the splash latitude, they do present a Kp dependent equation for the peak latitude of the drizzle, which is similarly related to the non-Kp version of the peak drizzle latitude as follows:

$$R = 35.5Kp - 33.1$$  \hspace{1cm} (2.13)

A Kp dependent equation for the peak latitude of the splash precipitation, $\lambda'_m$, is derived by substituting Equation 2.13 into the Foppiano and Bradley's non-Kp
Figure 2.2: Plot of $Q_1$ for a signal propagating from (38,266) to Churchill for several values of Kp.

dependent formula for $\chi_m'$ which yields:

$$\chi_m' = 68.3(1 - 0.021KP) + 0.18(1 + 0.7KP)|t|$$  \hspace{1cm} (2.14)

The formula for $\chi_m'$ will be fit to our data in Chapter 5.

The short term Foppiano and Bradley model predicts an aurora that spans a wider range of latitudes and has a more southerly peak for increasing values of Kp. This results in greater values of $Q_1$ as the value of Kp increases for signals received at the more southern auroral regions as shown in Figure 2.2. Universal time is placed on the independent axis and $Q_1$, expressed as a percentage, is on the dependent axis.
Chapter 2: Predicting the Path Loss of HF Radio Waves

Analysis regarding use of Kp in describing auroral absorption will be conducted in Chapter 5, where data will be compared to both the Kp and non-Kp parts of the Foppiano and Bradley model.

2.2.4 Auroral Absorption at Oblique incidence

Although Foppiano and Bradley give a relatively simple formula for calculating the vertical incidence absorption of a 30 MHz signal, calculating the oblique incidence of a signal of an arbitrary frequency is more complicated. The formula that Foppiano and Bradley use is

\[ L_{ob} = \frac{A_m(30 + f_i)^2 \Phi(f_v/f_{oE}) \sec i_h}{(f_{ob} + f_i)^2} \]  (2.15)

where \( L_{ob} \) is the loss in decibels due to absorption at oblique incidence, \( A_m \) is the median absorption of a 30 MHz signal at vertical incidence, \( f_i \) is the electron gyrofrequency about the vertical component of the Earth’s magnetic field, \( f_v \) is the equivalent vertical incidence frequency of a wave at oblique incidence, \( f_{oE} \) is the critical frequency of the E region of the ionosphere, \( i_h \) is the zenithal angle of the unrefracted oblique ray at a height of 100 km, and \( f_{ob} \) is the oblique frequency. This formula is based upon the model for calculating oblique incidence of HF radio waves proposed by George and Bradley [1974]. George and Bradley give a relationship for finding the equivalent vertical incidence frequency, \( f_v \), from the oblique frequency, \( f_{ob} \), as follows:

\[ f_v = f_{ob} \cos i_h \]  (2.16)

where \( i_h \) is:
Figure 2.3: The \( \Phi(x) \) function as given by George and Bradley [1974] where \( x = f_v / f_{0E} \).

\[
i_h = \arcsin(0.985 \cos \delta)
\]  \hspace{1cm} (2.17)

and \( \delta \) is the angle of takeoff for the signal. The electron gyrofrequency, \( f_I \), is calculated using:

\[
f_I = 2.8 \times \frac{2M \sin \lambda}{r^3}
\]  \hspace{1cm} (2.18)

where \( M \) is the magnetic moment of the earth approximated to be \( 7.82 \times 10^{19} G m^3 \), and \( r \) is the radius of the earth, \( 6.37 \times 10^6 \) meters.

The function \( \Phi(\frac{f_v}{f_{0E}}) \) is given by George and Bradley [1974] and is illustrated in Figure 2.3. The \( \Phi \) function approaches 1 for values of \( f_v \), the equivalent vertical incidence frequency, that are much greater than \( f_{0E} \), the critical frequency of the E region. This is true since signals of sufficiently high frequencies are able to penetrate the D region without deviation. The critical frequency of the E region of the ionosphere, \( f_{0E} \), is therefore a critical, though unmeasured, quantity. It is given by Davies [1965] for daytime hours as:
\[ f_{oE} = .9[(180 + 1.44R) \cos \chi]^{.25} \]  

(2.19)

where \( \chi \) is the solar zenith angle.

However since there is an E region present in the auroral regions of the ionosphere at night which is dependent upon auroral activity, we have modified Davies’ formula to take this into account as follows:

\[ f_{oE} = .9[(180 + 1.44R) \cos \chi]^{.25}(C_{foE} \times Q_1) \]  

(2.20)

where \( C_{foE} \) is an arbitrary constant to be fit to the data. We expect that \( C_{foE} \) will be around 0.1, since this would yield a value of \( f_{oE} \) of 1 MHz for the relatively high \( Q_1 \) value of 10 and lower \( f_{oE} \) values otherwise.

### 2.2.5 Daytime Ionospheric Absorption at Oblique Incidence

During the daytime the D region density is very high due to photoionization. This results in high ionospheric absorption in the D region and is independent of auroral effects. The daytime absorption is primarily a function of \( \chi \), the solar zenith angle, and is very large during the daylight hours, though is nonexistent at night. Davies [1965] gives the following expression for the daytime absorption, \( L_n \):

\[ L_n = \frac{430W(1 + .0035R) \cos^{.75} \chi \sec \phi}{(f_{ob} + f_t)^2} \]  

(2.21)

The smoothed sunspot number dependence, \( R \), is included as Davies shows that high daytime electron density profiles are correlated with a high sunspot number. The amount of absorption is also dependent upon the distance the wave travels within the D region of the ionosphere. Since the D region is relatively thin (15 km) as compared
to the height of the ionosphere, the distance the wave travels through this region can be approximated by the secant of the angle of incidence, \( \phi_D \). Davies also points out that absorption tends to be higher during the winter months due to higher electron densities. He gives \( W \) as the correction factor for this anomaly and is equal to 1.5 for the months of December and January, and 1.2 for November and February. The parameter, \( f_{ob} \), is the observed frequency, and \( f_l \) is the electron gyrofrequency.

The dependence upon the cosine of the solar zenith angle, \( \chi \) is the most intuitive and is given by:

\[
\cos \chi = \sin \lambda' \sin d + \cos \lambda' \cos d \cos h
\]  

(2.22)

where \( \lambda' \) is the geographical latitude, \( d \), the solar declination, is given by (Aurora):

\[
d = 23.45 \sin \left[ \frac{360}{365} (284 + \text{day#}) \right]
\]  

(2.23)

and \( h \) is the local hour angle of the sun measured with respect to apparent noon and is given by:

\[
h = 15|l_t - 12|
\]  

(2.24)

where \( l_t \) is the local time.

### 2.2.6 Spatial Attenuation

As a radio wave propagates, its power will be weakened proportionally to the square of the distance traveled. The simulation therefore calculates the spatial attenuation, \( L_a \), of a radio wave in decibels to be:
Chapter 2: Predicting the Path Loss of HF Radio Waves

Figure 2.4: Plot of path loss in decibels for a signal propagating from (38,266) to Churchill for each value of Kp, taking into account oblique incidence and other sources of loss. It can be seen that sunrise occurs around 14 UT and sunset occurs around 23 UT.

\[ L_a = 22 + 20 \log \left( \frac{Df_{ob}}{0.3} \right) \]  

(2.25)

where D is the distance traveled in km and \( f_{ob} \) is the oblique frequency of the signal in MHz (Austin Radio Labs), (Davies 1965).

2.2.7 Loss Due to Ground Reflections

The simulation estimates a 3 decibel loss for each time the signal reflects off the ground. Although Davies gives a more sophisticated model of ground loss, 3 decibels is a reasonable approximation for a 4 MHz signal at relatively low radiation angles.

Taking into account absorption at oblique incidence as well as spatial attenuation and loss due to ground reflections, results in the plot of absorption for Churchill as shown in Figure 2.4 where universal time is on the independent axis and absorption
Figure 2.5: Diagram showing the virtual path of a signal as compared to its actual path. The angle of incidence upon the ionosphere, $\phi_d$ is also shown (Davies 1965).

in decibels is on the dependent axis.

2.3 Path Analysis

2.3.1 Single path, Single Transmitter

In order to calculate the total path loss for a one transmitter system it is first necessary to calculate which paths the signal propagates along. Although the signals will in reality be curved due to the curvature of the ionosphere, the simulation calculates the signal’s virtual path, which represents the path the signal would take if it were not curved by the ionosphere.

Though the virtual reflection height varies by location, season and time of day, the simulation estimates it to be 350 km. We investigated other possible values for the virtual reflection height and found that altering it had little effect compared to other variables in the simulation.

Absorption mainly occurs in the comparatively thin D region of the ionosphere whose height is taken to be 85 km. In order to calculate the total absorption for a given path, the simulation computes the coordinates of each incidence of the D region, in corrected geomagnetic (CGM) coordinates, which is used to calculate the
Chapter 2: Predicting the Path Loss of HF Radio Waves

Figure 2.6: Diagram showing a single hop path from a transmitter, T to a receiver, R. The points of intersection upon the D region are used to calculate the auroral absorption. The virtual reflection height is shown as \( h' \) while the height of the D region is \( h_D \) (Davies 1965).

auroral absorption based on the Foppiano and Bradley model as well as the daytime absorption. To calculate the total loss for the path, the absorption contributions for each point of incidence upon the D region are summed together with losses resulting from spatial attenuation and ground reflections. For the purposes of calculating the absorption at oblique incidence, the incidence angle of the signal on the D region, \( \phi_D \), is also calculated and shown in Figure 2.6.

2.3.2 Multiple Hops, Single Transmitter

A radio wave can propagate from the same transmitter to the same receiver via several different paths as shown in Figure 2.7. For the purposes of the simulation it is assumed that the signal can reflect off the ionosphere up to three times and that the height of reflection is always the same. Signals that undergo four or more ionosphere reflections were found to be too weak to significantly impact the system. This has been verified by allowing more reflections in test cases and noting the error which occurred in reducing to three reflections. Furthermore, the points of reflection are
Figure 2.7: The one hop mode, two hop mode and three hop mode which the simulation calculates.

assumed to be evenly spaced with respect to the transmitter and receiver.

Much of the time, radio waves will tend to primarily propagate along the 1 hop mode, although the 2 hop and 3 hop modes are often significant. The simulation calculates the net loss for the system of paths involving 1 to 3 hops using the inverse sum equation as given by Davies [1965] as follows:

\[ L_r = -10 \log \left( \frac{1}{10^{0.1 \times L_1}} + \frac{1}{10^{0.1 \times L_2}} + \ldots \right) \]  \hspace{1cm} (2.26)

where \( L_r \) is the net system loss (in dB) and \( L_1, L_2, \ldots \) are the losses for each of the paths (in dB).

The inverse sum equation can be derived from the requirement that the net power received be equal to, in power units:

\[ P_r = P_t \left( \frac{1}{l_1} + \frac{1}{l_2} \ldots \right) \]  \hspace{1cm} (2.27)
and the path loss in decibels, $L_r$:

$$L_r = -10 \log \frac{P_r}{P_t}$$  \hspace{1cm} (2.28)

where $P_r$ is received power, $P_t$ is the transmitted power, and $l_1, l_2 \ldots$ is the power loss of each path. The above equations all utilize the relation between power units and decibels,

$$L = 10 \log l$$  \hspace{1cm} (2.29)

where $L$ is in decibels and $l$ is in power units of $(V/m)^2$.

For example, assume that a single transmitter emitted a signal that propagated along the 1 hop, 2 hop, and 3 hop paths resulting in a 40 dB loss, 40 dB loss, and 50 dB loss respectively. The corresponding losses in power units are $10^4$, $10^4$, and $10^5$. This results in a power loss $P_r/P_t$ of $1/10^4 + 1/10^4 + 1/10^5 = 2.1 \times 10^4$. Converting this into decibels yields a net loss of $-10 \log(2.1 \times 10^{-4}) = 36.8$ dB.

### 2.3.3 Multiple Transmitter System

The receivers receive signals from many different transmitters located throughout the United States and southern Canada. The simulation placed simulated transmitters on a grid located on top of this region and spaced 4 degrees in latitude and 10 degrees in longitude. The simulation calculated the loss for each transmitter receiver system, and computed the total, multiple transmitter, system loss by resolving the individual single transmitter systems using the inverse sum formula of the previous section. The results of the full simulation, using a simulated grid of transmitters are shown in Figure 2.8 with universal time on the independent axis and absorption in decibels on
Table 2.1: The geographical coordinates of the simulated transmitters.

The overall path loss is less in the multiple transmitter system than in the single transmitter system. In the multiple transmitter system there are some transmitters that are closer to the receiver and therefore can dominate since their signals will incur a smaller loss due to spatial attenuation. Also since the transmitters in the multiple transmitter system occupy a variety of longitudes, there is not as sharp an increase in path loss at local sunrise. These plots will later be compared to data for sites situated in southern Canada in order to test and improve the Foppiano and Bradley model.
Figure 2.8: Plots of the full simulation of Path loss for Churchill. The top plot show the model using the non-Kp dependent model, and the bottom model takes into account the Kp dependence.
Chapter 3

Simulation Design and Implementation

3.1 Overview of the Simulation

In the Chapter 2 we presented a model for predicting auroral absorption at oblique incidence and a method for predicting the possible propagation paths of a radio signal. Software was developed to simulate the net absorption suffered by radio waves propagating from multiple transmitters along different paths to a single receiver. The simulation was necessary since the actual locations of the sources of the signals received by our transmitters is not known.

Figure 3.1 is a detailed schematic of the execution of the simulation. There are three sets of inputs: the physical parameters to simulate, the values of the model's variables which will later be fitted, and the geometry of the simulated transmitters. Using the inputs the simulation determines the possible paths for each signal by calculating the points of incidence on the D region of the ionosphere. For each path the simulation then calculates the total absorption using the model presented in Chapter 2.
Figure 3.1: The overall design of the simulation. Ovals represent input/output and the rectangles represent program execution.

and determines the net loss for all of the different paths using the inverse sum equation also presented in that chapter. It is important that the simulation be designed to be not only be reasonably fast when it is executed once, but also to be as efficient as possible when it is executed many times in succession, since this will be required when attempting to fit the model to our data. Several executables were created to run different aspects of the simulation, under different circumstances. These executables include simplot, obliqueplot, multitransmit_plot, and benchmark_plot, which are all documented in Appendix C. We will focus on implementation of the simulation used by benchmark_plot, which is the program that fits the simulation to the data.

3.2 Inputs and Output

The three sets of inputs to the simulation originate from different parts of the software and are read into different data structures. The physical parameters are entered on the command line by the user to describe the receiver and a typical day. These parameters include the geographical location of the receiver, the average smoothed
sunspot number, R, the geomagnetic index, Kp, the day number, the average oblique frequency of the received signal, $f_{ob}$, and a typical virtual reflection height for the received signals. The user may also specify the geometry of the simulated transmitters by editing the file “transmitter.dat”. Each line of this file represents a single virtual transmitter, and contains the geographical latitude in degrees north of the equator, and the geographical longitude in degrees east of Greenwich, England. The last inputs are the variables of the model that are to be fitted to the data. These are passed in as a structure and represent a single node of the search space seen by the fitting module of the software. The simulation outputs the net absorption in decibels for each universal time interval which can later be plotted or directly compared to the data.

3.3 Calculating the Signal Paths

For each of the virtual transmitters the simulation calculates the points of incidence upon the D region for several possible paths. The paths calculated include the one hop mode which has two incidence points, the two hop mode which has four incidence points, and the three hop mode which has six incidence points. The geographical coordinates of each of the incidence points are calculated using spherical geometry equations, which were provided courtesy of S.E. Milan. Each of the incidence points are then converted into corrected geomagnetic coordinates using FORTRAN code also supplied courtesy of S.E. Milan. Converting between coordinate systems is especially slow since the simulation makes a system call to the conversion software. The twelve incidence points for signals propagating from a specified virtual transmitter are stored into a 2 dimensional propagation path array. One dimension corresponds to a specific mode and the other dimension corresponds to the D incidence points of a signal.
Figure 3.2: A schematic representation of the path propagation data structure for signals propagating from a single transmitter for the 1 hop, 2 hop, and 3 hop modes. propagating along that mode. In addition to the array of D incidence coordinates it is also necessary to store an array of takeoff angles upon the D region for each mode and the geographical distance from the virtual transmitter to the receiver. The propagation path data structure represents all the properties of signals propagating from a single virtual transmitter to a receiver and is shown in Figure 3.2.

For a single execution of the simulation it is only necessary to store one propagation path array in memory since it is possible to calculate the net absorption of the multiple transmitter system by maintaining a running total of the inverse of the power loss. However in order to fit the model to the data, it is necessary to run the simulation multiple times using different sets of values for the model’s variables. In order to do this efficiently the propagation path arrays for all the virtual receivers
need to be stored into memory since it is too time consuming to recalculate the signal paths each time the fitting algorithm runs the simulation. To accomplish this a dynamic array is used to store the propagation path structures for each of the virtual transmitters into memory.

3.4 Calculating the Net Loss

As stated in the Chapter 2 the simulation takes into account four types of path loss including, loss due to ground reflections, spatial attenuation, absorption due to photoionization and auroral absorption. The four contributions are summed together to obtain a total loss for a given path. The first two contributions to ground loss can be calculated without regard to where the signal enters the D region of the ionosphere. These two contributions can therefore be calculated for any mode using only the geographical distance calculated from transmitter to receiver and the array of takeoff angles using the equations presented in the Chapter 2. Calculating the absorption due to photoionization and auroral absorption for a mode requires applying the model discussed in the previous chapter to each of the points of incidence and summing them together. The path properties data structure illustrated in Figure 3.2 facilitates the calculation of the total absorption for any mode. The points of incidence and the takeoff angle for the mode can be extracted from the corresponding index number in the incidence point array and the takeoff angle array of the data structure. From the total losses of each of modes it is easy to apply the inverse sum formula discussed in Section 2.3.2 to calculate the net loss of signals propagating from any virtual transmitter. Calculating the net absorption for the entire multiple transmitter system is also relatively simple using the array of propagation paths. The simulation calculates the net loss for each propagation path structure in the array and applies the
inverse sum equation to derive the net system loss in the same way it calculates the net loss for the modes of any single transmitter. The net system loss is calculated for each universal time hour interval and output as either a postscript plot, to a datafile, or to the heuristic module of the fitting software.

3.5 Time Analysis

The simulation was compiled and tested on a computer with an Intel 200 MHz Pentium processor running the Linux operating system. Since user loads vary on the computer the execution times also vary, and are therefore only approximations.

When running the simulation multiple times there is an initial setup time to calculate the propagation paths, in addition to the time required to calculate the net system loss for each universal time interval. The setup depends linearly on the number of transmitters in the geometry. For a 28 transmitter system, the setup takes approximately 20 to 30 seconds on our computer to complete. This step takes some time since it involves system calls to convert between geographical coordinates and geomagnetic coordinates. However the software data structures in the software design allow the fitting algorithm to run the simulation multiple times while only calculating the propagation paths once. The time it takes to calculate the net system losses is more critical since this part of the simulation is executed many times by the fitting software. Since this part of the software does not need to recalculate the propagation paths it requires only about a second for each iteration. This is important since the time for this step dictates how fast the software will be able to fit the model to the data.
Chapter 4

Data Collection and Analysis

4.1 Introduction

The goal of the data collection is to test experimentally the Foppiano and Bradley model. Plots of average signal strength in decibels vs. universal time were created from radio receivers for the frequency range of 3.6 MHz to 4.4 MHz. This frequency range is in the HF frequency band which experiences substantial absorption due to auroral precipitation. This band was also selected since it is in the domain of applicability of the Foppiano and Bradley model. The receivers are stationed on the same longitude in the southern portion of the auroral oval to study the latitude dependence of the aurora. The plots of average signal strength can then be compared qualitatively to the Foppiano and Bradley model, and later quantitatively to the output of the simulation based in part on that model.

Foppiano and Bradley give different possible formulas for the latitude dependence of the aurora. One is based on the smoothed sunspot number, R, which they conclude is valid for predicting long term trends, and the other is based on the geomagnetic activity, Kp, which they conclude is a better short term predictor. Both of these possibilities will be explored qualitatively in this chapter.
4.2 Instrumentation

The receivers used in the study are programmable stepped frequency receivers which collect data for the frequency ranges of 30 kHz to 5 MHz in 10 kHz steps with a 2 second time resolution and have been operating in northern Canada since the fall of 1994. The receivers use a magnetic field loop antenna with an area of 10m² to measure the intensity of the incoming signals.

The field strength of the incoming signal can be deduced by measuring the fluctuating current in the antenna. The electronics logarizes the power spectral density in each 10 kHz and then digitizes 8 bits deep. This is stored to disk and then mailed to Dartmouth to be archived.

The Canadian receivers used in this study are stationed roughly on the same longitude (265°E) and at a range of latitudes as shown in Figure 4.2: Data from Gillam, Churchill, and Arviat were primarily studied since these sites have been
Figure 4.2: The locations of the HF radio receivers operated by Dartmouth College in the northern hemisphere, and the location of the auroral oval, (Shepherd 1998).

<table>
<thead>
<tr>
<th>Site</th>
<th>ID</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Mag Lat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gillam</td>
<td>GI</td>
<td>56.38 N</td>
<td>265.36 E</td>
<td>66.82 N</td>
</tr>
<tr>
<td>Churchill</td>
<td>CH</td>
<td>58.76 N</td>
<td>265.92 E</td>
<td>69.14 N</td>
</tr>
<tr>
<td>Arviat</td>
<td>AR</td>
<td>61.11 N</td>
<td>265.95 E</td>
<td>71.35 N</td>
</tr>
<tr>
<td>Baker Lake</td>
<td>BL</td>
<td>64.32 N</td>
<td>293.97 E</td>
<td>74.15 N</td>
</tr>
<tr>
<td>Taloyoak</td>
<td>TA</td>
<td>69.54 N</td>
<td>266.45 E</td>
<td>79.12 N</td>
</tr>
</tbody>
</table>

Table 4.1: The geographical and geomagnetic coordinates of the HF receivers.

active since 1994 and have been relatively reliable.

Although the sites continuously record data throughout the year, this study has focused on data in the 90 day range around the winter solstice of days 305-30. A ninety day range was used as it renders good statistics while still maintaining a seasonal consistency. One of the major advantages of concentrating on the data near winter solstice is that it maximizes the number of nighttime hours, which is when the absorption due to the splash precipitation occurs. The daylight hours in the data are not useful since the absorption is dominated by the massive ionospheric absorption
resulting from the photoionization in the D region. Furthermore, utilizing late fall – early winter data minimizes the number of thunderstorms which often appear in the northern hemisphere from April through October. The storms can cause the lower HF band to be dominated by atmospherics which are more varied and can have entirely different source locations than the more consistent man-made signals that normally dominate this band. Data were inspected visually and nights on which significant thunderstorm activity occurred were removed from the set. Data from four different winter solstices were studied at the Gillam, Churchill, and Arviat sites for four years of data corresponding to the winter season of 1994 through the winter season of 1997.

4.3 Data Format and Manipulation

Figure 4.3 is an example of a full day of data from one of the Canadian sites. Universal Time is on the horizontal axis, and frequency in kHz is on the vertical axis. The intensity of the received signal is represented by the grayscale, where black pixels represent high received intensity and the white pixels represent low received intensity. The dark horizontal streaks represent man-made transmissions. It should also be noted that local midnight corresponds to 0630 UT.

The frequency range of 3600 kHz to 4500 kHz, which represents the lower end of the HF frequency band, was the primary focus since the Foppiano and Bradley model of absorption at oblique incidence is based on HF signals. However, the 420-440 kHz band was used to screen out bad data, and the 1210-1250 kHz band was also used to distinguish polarization at Churchill for most of 1997 and the first part of 1998. The frequencies below 1500 kHz are much stronger since they are dominated by AM transmissions. Some of these transmissions are local and are not affected by ionospheric absorption. The frequencies above 1500 kHz are not local and exhibit
Figure 4.3: A typical grayscale plot for Churchill.

significant propagation effects, such as the obvious day-night effect.

Figure 4.4 is the integrated signal strength from December 17, 1997 of Churchill, which is the same day as the grayscale plot of Figure 4.3 for the frequency range of 3600 kHz to 4500 kHz. There are no data points from 16 UT to 19 UT since no data was taken during this time. The large peak from 11 UT to 13 UT is consistent with the dark streaks from man-made transmissions that occur in the grayscale during this time frame. Likewise the minimum at 5 UT is consistent with the very light section that appears data between 4 and 7 UT at the specified frequency range.

The integrated power in $(V/m)^2$ is calculated from the raw data as a function of universal time for the frequency range of 3600 kHz to 4500 kHz. This is done by squaring the field strength, which is calculated from the intensity using the measured calibration relation, multiplying the result by the bandwidth (10 kHz) and summing
Figure 4.4: The signal strength received at Churchill on December 17, 1997.

over the frequency range. To save memory and computing time the files are compressed by averaging the integrated power in 5 minute bins. The integrated power is converted to power decibels as follows:

\[ S = 105 + 10 \log P_r \]  

(4.1)

4.4 Statistical Analysis of the Data

Multiple days of data are combined by tabulating the number of instances of each received decibel level, from 20 dB to 65 dB, for each 5 minute period in the day range. In practice the receivers seldom recorded a signal strength of over 60 dB. This is done for each of the 1 hour bins consisting of 20 UT through 15 UT. The missing hour ranges occur over local noon when there is a very high level of ionospheric
absorption and thus would result in very weak received signals. A statistical average of the integrated powers of the corresponding decibel levels is then calculated from the occurrence tables and plotted as a function of signal strength in decibels vs. UT. For Churchill, local time is six and a half hours behind universal time.

![Graph showing signal strength vs. time](image)

**Figure 4.5:** The average signal strength received at Churchill for the period of December 11, 1997 to March 11, 1998.

The 90 day ranges yield good statistics as the graphs are relatively smooth without too many exaggerated spikes. Figure 4.5 depicts the average integrated signal strength for Churchill for a 90 day range around the 1997-1998 winter solstice. The signal strength drops off rapidly around 15 UT, which coincides roughly with local dawn, and rises sharply around 21 UT, which is local dusk. This is consistent with the fact that the daytime ionospheric absorption is very high. The slight drop in signal strength around 6 to 8 UT is most probably due to auroral absorption. This is to be expected since the dip occurs around local midnight, which is when the nighttime absorption due to the splash precipitation peaks. By studying data from the three
Table 4.2: Winter solstice data of average received signal strength for each of the sites Arviat, Churchill, and Gillam from 1994-5 to 1997-8. The plots are of signal strength vs. universal time with local midnight falling around 0630 UT.

different sites over four years it becomes possible to examine some interesting trends in the data. Unfortunately, due to receiver problems, especially during the winter of 1996/1997, some periods of data at some of the sites were not available.

Table 4.2 shows the average HF integrated power vs. universal time for 3 sites
Table 4.3: The smoothed sunspot number, R, and average index of geomagnetic activity for each interval studied.

over a four year period for the interval of November 1 to February 4. Because of the lack of November data for 1997, the 1997-1998 data represents the interval of December 10 through March 10. Each of these plots is identical in format to Figure 4.4 and the panel corresponding to Churchill 1997/1998 is exactly the same. The splash auroral absorption in each of the plots can be seen as a drop in signal strength during the hours after local midnight, 6 to 10 UT. As expected, the amount of auroral absorption is higher for Arviat and Churchill than for Gillam, which for quiet times is just south of the auroral oval. A more interesting trend is that the amount of auroral absorption for the 1997-1998 solstice plots is much lower than for the other winter seasons even though the smoothed sunspot number, R, is higher for this interval than for the other intervals. Qualitatively, this is contrary to the expected output of the long term Foppiano and Bradley model, which models the auroral oval as being more southerly and larger, for higher sunspot numbers, which would result in greater absorption values for higher sunspot numbers. One possible explanation for this is that Kp, an index of geomagnetic activity, is a better predictor of auroral activity. Although higher values of Kp tend to be correlated with higher sunspot numbers, this is not always case. It happens not to be the case for the 1997-1998 winter season, which saw a much lower value of Kp than the 1994-1995 winter solstice, even though its sunspot number was significantly higher.
4.5 Including Kp

The Kp index reflects the solar wind effects on the earth’s magnetic field and is therefore likely to be a good predictor of auroral activity. The Kp index is updated every three hours and reported by National Geophysical Data Center at ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC_DATA/INDICES/KP_AP. Foppiano and Bradley also concluded that the Kp index was a better predictor of auroral activity for short term predictions though they retained the solar epoch as a predictor for long term predictions. In order to test whether Kp is a good short term predictor, the integrated plots were created for the 1997-1998 winter solstice at Churchill with the data being sorted by Kp.

![Graph of Kp index vs. time](image)

**Figure 4.6:** The 1997-8 Churchill data sorted by the Kp index.

Figure 4.6 qualitatively shows that higher values of Kp will result in greater auroral absorption. The signal strength around 6-10 UT dips much more sharply for the Kp values of 2, 3 and 4 than for the Kp value of 1, which shows only a slight dip in signal strength as a result of auroral absorption. The curves corresponding to the Kp values
of 2, 3, and 4 are much less smooth than for the curve corresponding to the Kp value of 1. This is due to the fact that the overall geomagnetic activity during this time period was relatively low and resulted in poorer statistics for the higher Kp values. In Chapter 5 we will attempt to fit the Foppiano and Bradley model to the data shown in this chapter and further explore the Kp dependence of auroral absorption. Milan, et al. [1995] saw a similar effect on a 6.8 MHz signal, which motivated us to do this analysis in the 3.6 to 4.5 MHz range.
Chapter 5

Fitting the Model to the Data

5.1 Introduction

In Chapter 4 the data were presented and compared qualitatively to the results of the simulation, which was discussed in Chapter 2. In this chapter we quantitatively compare the data to the simulation and attempt to adjust the Foppiano and Bradley model to better describe our data. Since the data for the 1994/1995 winter solstice and the 1997/1998 winter solstice were more complete than for the other years, only these two intervals will be fit to data. A least squares error was calculated between the model and the data for nighttime hours in order to quantitatively estimate how well the model fit the data. A computational search was then conducted in order to find the values for the variables in the model that would minimize the error. There are obviously too many variables in the model to fit them all, so only eight of the most critical were analyzed. Since it was shown in Chapter 4 that the data appeared to be qualitatively dependent upon the geomagnetic index, Kp, variables from both the Kp dependent and non-Kp dependent parts of the Foppiano and Bradley model will be fitted. We initially do not include Kp, then extend the analysis to include Kp.
5.2 Determining the Goodness of Fit

Before the goodness of fit between the data and the simulation can be calculated it should be noted that the simulation cannot be directly compared to the data since it represents overall path loss in decibels while the data plots represent received signal strength in decibels. In theory this could be resolved by subtracting the simulation’s path loss curve from the net signal strength transmitted, which could then be compared to the data.

\[ S_{\text{received}} = S_{\text{transmitted}} - S_{\text{loss}} \]  

(5.1)

The difficulty that arises is that the net transmitted signal strength is not a known quantity. One possibility would be to subtract the data from a ‘quiet day’ curve as is often done when analyzing riometer data. However, we decided to leave the data in the relatively raw form and manipulate the simulation output instead to match it. This makes sense since everything is ‘known’ in the simulation. Using the simulation output, we estimated the transmitted signal strength in decibels by adding the average of the signal strength received to the average path loss as estimated by the simulation. This is a reasonable approach since it is the change of signal strength received during the night that will yield insights into the absorption effects of the aurora rather than the absolute signal strength received.

The goodness of fit between the data and the model’s output for the parameters corresponding to the data was calculated using the standard error of estimate as follows (Lapin 1973):

\[ \text{Error} = \sqrt{\frac{\sum(Y_{\text{data}} - Y_{\text{model}})^2}{n - 1}} \]  

(5.2)
where $Y$ is the signal strength in decibels for each universal time interval that was analyzed and $n$ is the number of universal time intervals which were analyzed. Since this thesis is concerned only with nighttime auroral absorption, the universal time hours of 0 through 14 were fit to data. One is subtracted from the variable $n$ in Equation 5.2 since the transmitted signal strength is estimated from the data and not calculated in the model.

Physically, the standard error represents how close (in decibels) the data are to the results of the simulation. Statistically, 68% of the data points should be within the standard error of estimate to the simulation. The standard error therefore provides a good means of analyzing how well the simulation represents the data.

### 5.3 Searching for the Best Fit

The search space of possible parameter values is multidimensional where each dimension corresponds to one of the variables to be fit to data. The set of possible values for any variable in the search space can be derived from the starting value, and from a precision constant, $\delta$, which dictates the spacing between neighboring nodes in the search space of that variable. For example, a variable assigned a starting value of 10 with $\delta = 2$ would have a search space consisting of the set of even numbers with 8 and 12 being its nearest neighbors. Although this search space has an infinite number of nodes it could still be used since it is implicit in that it is not stored in memory. However for time considerations the space was confined to only the physically reasonable values of the variables. In our case this results in a number of possible values for each of the variables to be around 10.

Since the search space is extremely large, ($\sim 10^8$ in the case of eight variables), it is not feasible to search every possible combination of values for the set of variables
to be fitted. Therefore, a search algorithm was implemented in order to find the values of the variables that would yield the good fit for the data, and still finish in a reasonable amount of time. The algorithm implemented was a hill-climbing search given below as a recursive function (Russel, 1995):

\[
\text{Hill-climbing}(\text{current}_\text{node}) \\
\quad \text{next}_\text{node} = \text{Best}_\text{Heuristic}(\text{nearest}_\text{neighbors}(\text{current}_\text{node})) \\
\quad \text{if } \text{Heuristic}(\text{next}_\text{node}) > \text{Heuristic}(\text{current}_\text{node}) \\
\quad\quad \text{return } \text{current} \\
\quad \text{else} \\
\quad\quad \text{return } \text{Hill-climbing}(\text{next}_\text{node}) \\
\text{endif}
\]

The starting point of the search was the set of values that corresponded to the Foppiano and Bradley model. The heuristic function used in the hill-climbing search is the standard error estimate between the data and the output curve of the simulation.

The hill-climbing algorithm works by trying to constantly improve on the current state by looking to see if any of the neighboring states is better. The program runs the simulation for each of the neighboring states and jumps to the state which has the smallest error with the data. The implementation permits travel in search space along only one dimension at a time, and thus it defines the nearest neighbors as the sixteen cardinal points in an eight dimensional search space. The search algorithm could be modified to look at the diagonal points in attempt to climb through the search space with fewer iterations. However in this search space the vast number of diagonal point causes the search to be much slower, since each check of a neighboring point requires running the simulation.

One of the drawbacks with a hill-climbing approach is that it can return a local minimum, a situation where the neighboring states have greater errors, yet there may be a nearby state that is even smaller. One way to attempt to find the overall best
Table 5.1: Sample search for Arviat 1994-5. Each iteration adjusts one of the variables to improve the error. The bottom line represents the best fit for this search.

state in the search space is to repeat the algorithm several times with different initial states. Fortunately, the search space was such that the hill-climbing algorithm yielded very good results on the search and rerunning the algorithm using different initial states did not tend to greatly improve the results. Overall the algorithm efficiently fit the model to the data by rendering good fits while only needing to check a tiny fraction of the search space. Our problem is especially well suited to this approach, since the result of the FB model provides a logical starting point.

5.4 Analyzing the Search Algorithm

Although the hill-climbing algorithm is fast it is likely that it will not find the global minimum and hence the absolute best fit since it does not check the entire search space. The ability for the algorithm to find the global minimum is dependent on the nature of the search space. This algorithm will tend to yield better results on a search space with fewer local minima since there are fewer points in the search space that will cause it to stop before it has found the global minima. We hypothesized that the our search space would contain relatively few local minima since each variable
should have an optimal value which would be independent of values of the other variables in the simulation. To test both the optimality and the performance of the hill-climbing algorithm, we compared it to a comprehensive search algorithm which tests every possible set of values. Since the comprehensive search algorithm is much slower we used a simplified 3 variable search space of the non-Kp model to compare both algorithms. There were 700 possible combinations in the test search space.

<table>
<thead>
<tr>
<th>Var</th>
<th>Min</th>
<th>Max</th>
<th>δ</th>
<th>Num. Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{max}$</td>
<td>0.0</td>
<td>3</td>
<td>0.5</td>
<td>7</td>
</tr>
<tr>
<td>$T_{width}$</td>
<td>5</td>
<td>50</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>$s_{amp}$</td>
<td>6</td>
<td>24</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>$C_{f_{oE}}$</td>
<td>0.0</td>
<td>0.0</td>
<td>N/A</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 5.1:** The simplified search space used to compare the hill-climbing algorithm and the comprehensive search.

The best fits found using both algorithms are summarized in Table 5.2. As seen from this table the hill-climbing returns fits that are remarkably close to the overall best fit found using the comprehensive algorithm. In fact in half of the trials the hill-climbing algorithm returned the overall best fit. The largest discrepancy in the errors returned by both algorithms was 0.04 dB and the mean discrepancy was 0.02 dB. Thus the hill-climbing algorithm appears to produce fits that are about as good as the global best fit.

<table>
<thead>
<tr>
<th>Site/Yr</th>
<th>Comprehensive</th>
<th>Hill-climbing</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 94-95</td>
<td>1.02</td>
<td><strong>1.02</strong></td>
</tr>
<tr>
<td>CH 94-95</td>
<td>1.27</td>
<td>1.31</td>
</tr>
<tr>
<td>GI 94-95</td>
<td>1.12</td>
<td><strong>1.12</strong></td>
</tr>
<tr>
<td>AR 97-98</td>
<td>2.57</td>
<td>2.58</td>
</tr>
<tr>
<td>CH 97-98</td>
<td>2.52</td>
<td><strong>2.52</strong></td>
</tr>
<tr>
<td>GI 97-98</td>
<td>3.77</td>
<td>2.81</td>
</tr>
</tbody>
</table>

**Table 5.2:** The standard errors of the fits produced by the comprehensive search and the hill-climbing search algorithms. Entries in bold are when hill-climbing algorithm returned the absolute best fit.
Table 5.3 shows that the Hill-climbing algorithm is much more efficient than the comprehensive search. The speed of the search algorithm is largely determined by the number of points it checks and hence the number of times it needs to execute the simulation and compare it to the data. The hill-climbing algorithm was able to find excellent fits by analyzing only 54 combinations average or 7.7% of of the search space. This makes the hill-climbing approach over ten times faster than comprehensive method.

<table>
<thead>
<tr>
<th>Site/Yr</th>
<th>Comprehensive</th>
<th>Hill-climbing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Num Checks</td>
<td>Time (sec)</td>
</tr>
<tr>
<td>AR 94-95</td>
<td>700</td>
<td>703</td>
</tr>
<tr>
<td>CH 94-95</td>
<td>700</td>
<td>703</td>
</tr>
<tr>
<td>GI 94-95</td>
<td>700</td>
<td>703</td>
</tr>
<tr>
<td>AR 97-98</td>
<td>700</td>
<td>707</td>
</tr>
<tr>
<td>CH 97-98</td>
<td>700</td>
<td>753</td>
</tr>
<tr>
<td>GI 97-98</td>
<td>700</td>
<td>707</td>
</tr>
<tr>
<td>Average</td>
<td>700</td>
<td>713</td>
</tr>
</tbody>
</table>

Table 5.3: The number of nodes analyzed and running times of the comprehensive and the hill-climbing algorithms.

One major advantage of the hill-climbing approach is that it is very fast even when the size of the search space is much larger. When an eight variable search space consisting of approximately $10^8$ points is used the hill-climbing algorithm is still able return a good fit in a reasonable amount of time (5 to 10 minutes), where a comprehensive search would have taken over two years to complete, assuming about one second per check. This is due to the fact that the comprehensive search runs in exponential time based on the number of dimensions: $O(S^n)$, where $n$ in the number of dimensions of the search space and $S$ is the number of intervals per dimension. Although the running time of the hill-climbing search is dependent on the nature of the search space, we can estimate it for our search space, where it generally moved
in only direction along any dimension. Therefore for our search space the greatest
number of points to be analyzed would be proportional to $S \times n$. However, analyzing
whether a point is a minima requires checking all the nearest neighbors of that point,
which is equal to $2 \times n$ assuming the nearest neighbors are defined as the cardinal
points. Thus the hill-climbing algorithm runs in $O(Sn^2)$ time.

The hill-climbing algorithm could be made more effective in finding better fits by
using a random restart version which would randomly start elsewhere in the search
space when it encountered a minimum. One could also use a simulated annealing
algorithm which would allow the search to advance to worse fits at times in attempt
to find an even better fit in another nearby portion of the search space. However
these might slow down the search and do not seem necessary with our search space.
Therefore the hill-climbing algorithm will be used for all the fittings in this thesis.

5.5 The Search Spaces

5.5.1 The Variables of the Non-Kp Foppiano and Bradley
Model

The variables that will be fitted in the FB Non-Kp dependent model include $s_{amp}$,
the amplitude-scaling factor of the splash precipitation, two variables in the diurnal
dependence, and $C_{fOE}$, the constant in our formula for $f_{OE}$, which was presented
in Chapter 2. In trial runs we also varied the reflection height but found that this
variable played only a small role, so we set it to 350 km.

The amplitude-scaling factor of the splash contribution, which Foppiano and
Bradley give as 12, will be fit to the data and allowed to vary from 6 to 60 in in-
crements of 2. The diurnal contribution of the splash precipitation has been slightly
modified to study the diurnal Gaussian width and the local geomagnetic peak time.

\[ s_t = \exp \left[ -\frac{(t_2 - T_{\text{max}})^2}{T_{\text{width}}} \right] \] (5.3)

In the Foppiano and Bradley model, \( T_{\text{max}} \) is equal to 0, local geomagnetic midnight, and \( T_{\text{width}} \) is equal to 15.7. A minimum value for \( T_{\text{max}} \) of 0.0 is used as \( T_{\text{max}} \) does not tend to peak before local midnight but often peaks slightly later than local midnight. \( C_{f_oE} \) models the auroral contribution to the E region of the ionosphere, which can exist due to auroral activity. The critical frequency of the nighttime E region as given in Chapter 2 is:

\[ f_{oE} = C_{f_oE} \times Q_1 \] (5.4)

Since the nighttime \( Q_1 \) values range from about 0 - 6\% and the critical nighttime frequency of the \( f_{oE} \) region may be about 0 - 2 MHz, values of \( C_{f_oE} \) should typically be around 0.1 - 0.3. Table 5.4 summarizes the four Kp independent variables to be fitted, their Foppiano and Bradley values, and their bounds and precision constant used by the search algorithm.

<table>
<thead>
<tr>
<th>Var</th>
<th>FB Val</th>
<th>Min</th>
<th>Max</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{max}} )</td>
<td>0.0</td>
<td>0.0</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>( T_{\text{width}} )</td>
<td>15.7</td>
<td>5</td>
<td>55</td>
<td>5</td>
</tr>
<tr>
<td>( s_{\text{amp}} )</td>
<td>12</td>
<td>6</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>( C_{f_oE} )</td>
<td>N/A</td>
<td>0.0</td>
<td>0.5</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Table 5.4:** The non-Kp dependent variables to be fit to the model, their Foppiano and Bradley values along with the range and precision of the search space.
5.5.2 The Variables of the Kp Dependent Model

In the Foppiano and Bradley model, Kp is introduced for short term predictions of auroral absorption. This will introduce several new variables to be fitted to the data in addition to the variables described in the previous section. It was shown qualitatively from our data in Chapter 4 that auroral absorption is dependent upon Kp in both the short and long term. Foppiano and Bradley introduce Kp into the model by replacing the Gaussian width of the latitude dependence as follows:

\[ \sigma_\lambda = 2.16(1 + 0.17Kp) \]  \hspace{1cm} (5.5)

Comparing the two formulas in the model for \( \sigma_\lambda \) yields the following relationship between R and Kp:

\[ R = \sigma_a Kp + \sigma_b \]  \hspace{1cm} (5.6)

where \( \sigma_a = 30.6 \) and \( \sigma_b = -70.0 \) as given in the Foppiano and Bradley model.

Foppiano and Bradley do not give a Kp dependent formula for the peak latitude of splash precipitation, though they do give one for the peak latitude of the drizzle precipitation, which can be derived by a substitution of the form:

\[ R = \lambda_a Kp + \lambda_b \]  \hspace{1cm} (5.7)

It is therefore reasonable to derive a Kp dependent formula for the peak latitude of the splash contribution by substituting Equation 5.7 back into the standard Foppiano and Bradley formula for the peak latitude of the splash precipitation. The Kp dependent formula for the peak latitude of the splash contribution would thus become:
Table 5.5: The Kp dependent variables to be fitted with their Foppiano and Bradley values along with the range and precision of the search space.

<table>
<thead>
<tr>
<th>Var</th>
<th>FB Val</th>
<th>Min</th>
<th>Max</th>
<th>δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ_a</td>
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<td>25</td>
<td>45</td>
<td>2</td>
</tr>
<tr>
<td>σ_b</td>
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<td>-40</td>
<td>5</td>
</tr>
<tr>
<td>λ_a</td>
<td>N/A</td>
<td>25</td>
<td>45</td>
<td>2</td>
</tr>
<tr>
<td>λ_b</td>
<td>N/A</td>
<td>-90</td>
<td>-40</td>
<td>5</td>
</tr>
<tr>
<td>a</td>
<td>N/A</td>
<td>25</td>
<td>45</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>N/A</td>
<td>-90</td>
<td>-40</td>
<td>5</td>
</tr>
</tbody>
</table>

\[
\lambda'_m = 67(1 - .0006(\lambda_a Kp + \lambda_b)) + 0.3(1 + .012(\lambda_a Kp + \lambda_b)) |t| \tag{5.8}
\]

where \( \lambda_a \) and \( \lambda_b \) are variables to be fitted.

In addition we also tested an alternative model which replaces \( s_R \), the solar epoch dependence of the splash with \( s_{Kp} \), which represents a geomagnetic dependence for the splash precipitation as follows:

\[
s_{Kp} = 1 + .009(aKp + b) \tag{5.9}
\]

Equation 5.9 derives \( s_{Kp} \) from \( s_R \) in the Foppiano and Bradley model by substituting \( aKp + b \) for \( R \). For completeness, the geomagnetic activity dependence was derived for the drizzle using this same substitution for \( R \) in the \( d_R \) term. Although \( a \) and \( b \) are independent variables, we fixed \( a \) to be the average of \( \sigma_a \) and \( \lambda_a \), and likewise \( b \) to be the average of \( \sigma_b \) and \( \lambda_b \), in order to maintain a manageable size for the search space. The six Kp dependent variables to potentially be fitted, their Foppiano and Bradley values, their bounds, and precision constant used by the search algorithm are summarized in Table 5.5. In practice not all of these variables will be independently fit in any one search.
5.6 Results

5.6.1 Fits Without Including Geomagnetic Effects

We first attempted to fit the variables of the non-Kp dependent Foppiano and Bradley model to our data in order to determine how well the non-Kp dependent model described the data. The standard non-Kp dependent FB model, which uses their values for the non-Kp dependent variables, will be used as a baseline with which to compare all the fitted models. Although Kp is not included in this model, the smoothed sunspot number is included in both the latitude dependence of the splash term as well as in the solar epoch dependence, $s_R$. The results of this fitting for the 1994-1995 winter solstice, and the 1997-1998 winter solstice are shown below in Table 5.6.

These plots shown in Table 5.6 depict the data (solid line) and the two model results: the FB model (dotted line) and the best fit (dashed line). All four of the Kp independent variables were allowed to be fitted. The values of the variables that produced these best fits as well as both errors of the fitted model and the standard FB model are shown in Table 5.7.
Table 5.6: Plots of the best fit found (dashed curve) by varying all of the variables of the non-Kp model. The standard results of the Foppiano and Bradley model (dotted curve) are presented as a baseline and the data is the solid line.
Table 5.7: The values of the variables generating the best fit for the non-Kp model, their standard errors, and the standard errors produced by the Foppiano and Bradley model.

<table>
<thead>
<tr>
<th></th>
<th>AR 94-95</th>
<th>CH 94-95</th>
<th>GI 94-95</th>
<th>AR 97-98</th>
<th>CH 97-98</th>
<th>GI 97-98</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{amp}$</td>
<td>14</td>
<td>20</td>
<td>14</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>0.5</td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$T_{width}$</td>
<td>25</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$C_{foE}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.58</td>
<td>2.52</td>
<td>3.81</td>
</tr>
<tr>
<td>Error</td>
<td>1.02</td>
<td>1.31</td>
<td>1.12</td>
<td>2.69</td>
<td>2.66</td>
<td>3.88</td>
</tr>
<tr>
<td>FB Err</td>
<td>1.15</td>
<td>1.94</td>
<td>1.92</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table shows that the errors for the fitted models are significantly better for the 1994-1995 Gillam and Churchill data, though the fitted models for the other sites only feature slight improvements. The most important trend in the data is that both the standard model and the fitted model have significantly smaller errors for the 1994-1995 data than for the 1997-1998 data. The 1.3 - 1.4 dB difference in the errors of the models between the two years are visually apparent from Table 5.6. The smoothed sunspot number was significantly higher for the 1997-1998 solstice than for the 1994-1995 solstice, but the Kp index was much lower for the 1997-1998 solstice. One possible way to improve the fits for the 1997-1998 data would be to use the Kp index in the model as suggested qualitatively in Chapter 4.

### 5.6.2 Fits Including Geomagnetic Effects

Substituting a Kp dependence in both splash latitude peak and the width of the splash latitude Gaussian yields dramatic improvements in the fittings, especially for the 1997 - 1998 winter solstice, as shown in Table 5.8. With the exception of the variables $a$ and $b$, which were set to the average of $\sigma_a$, $\lambda_a$, and $\sigma_b$, $\lambda_b$ respectively, all of the other variables discussed in Tables 5.4 and 5.5 where allowed to be fitted within the specified range.
Table 5.8: Plots of the best fit found (dashed curve) by varying all of the variables of the Kp model. The standard non-Kp results of the Foppiano and Bradley model (dotted curve) are presented as a baseline and the data is the solid line.
Table 5.9: The values of the variables generating the best fit for the Kp model, their standard errors, and the standard errors produced by the Foppiano and Bradley model.

As shown in Table 5.9, the errors for the fitted Kp dependent model are significantly less than the errors produced by the standard model, and the fits for the 1997-1998 data are especially better. This suggests quantitatively that Kp is an important parameter in predicting auroral absorption and should be included in the model.

In order to gain more insight into how each of the variables influences the results and whether it is possible to isolate values for the eight variables, the fitting algorithm was run several times with a different combination of variables that were fixed based on previous results. The outcomes from these runs are summarized in the Table 5.10, with the variables that were fit in bold face and the variable fixed in plain face for each of the runs. Also included in this table is the error for each fitting and the error of the standard FB model. It should be noted in the five and six variable fits $\sigma_a$ and $\sigma_b$ were not set to specific values, but fixed to the values of $\lambda_a$ and $\lambda_b$, which were allowed to vary in the fitting.
<table>
<thead>
<tr>
<th>Site/Yr</th>
<th>Vars</th>
<th>$s_{amp}$</th>
<th>$T_{max}$</th>
<th>$T_{width}$</th>
<th>$C_{FoE}$</th>
<th>$\lambda_{a}$</th>
<th>$\lambda_{b}$</th>
<th>$\sigma_{a}$</th>
<th>$\sigma_{b}$</th>
<th>Error</th>
<th>FB Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR-95</td>
<td>8</td>
<td>14</td>
<td>0.5</td>
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<td>0.10</td>
<td>33</td>
<td>-75</td>
<td>31</td>
<td>-50</td>
<td>1.00</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>14</td>
<td>0.5</td>
<td>25</td>
<td>0.10</td>
<td>33</td>
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<td>33</td>
<td>-65</td>
<td>1.00</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>16</td>
<td>0.0</td>
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<td>0.20</td>
<td>31</td>
<td>-55</td>
<td>31</td>
<td>-55</td>
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<td>1.15</td>
</tr>
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<td>-80</td>
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<td>-80</td>
<td>1.02</td>
<td>1.15</td>
</tr>
<tr>
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<td>-80</td>
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</tr>
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<td>28</td>
<td>-80</td>
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</tr>
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Table 5.10: Table of all the fitting runs for the Kp model. Bold-face variable values are fitted and the other variable values are fixed.
Chapter 5: Fitting the Model to the Data

The diurnal peak of the splash, $T_{max}$, and the width of the diurnal Gaussian, $T_{width}$, could not be fixed as they exhibited definite trends across either site location or year, and changing them greatly affects the model, which was determined in trial runs.

The $T_{max}$ variable seems to exhibit a strong site dependence, as the splash seems to peak at 3 MLT at the southern most site of Gillam, but peaks at around 0.5 MLT at the northern site of Arviat. Although this would seem to imply that the aurora would peak at later times at more southern latitudes, it is also possible that this effect is due to the fact that the locations of the transmitters simulated by the program do not completely accurately represent the true distribution of transmitters. It is likely that this difference would affect the southern most sites more since they are potentially closer to the transmitters that are not accurately represented by the program. For example, placing more transmitters in the east would have the effect of moving the peak of the splash earlier and therefore closer to local midnight, though the magnitude of this effect is still to be determined.

The diurnal width of the splash Gaussian seems to be much wider in the higher Kp solstice of 1994-1995 data than for the 1997-1998 data, which suggests that it may be Kp dependent. However, determining a possible Kp dependence for the diurnal width, would require analysis of much more data, and is beyond the scope of this thesis.

Analysis of Table 5.10 shows that the three variable fit with the $C_{foE}$ fixed at 0.15, $\lambda_a$, $\sigma_a$ fixed at 28 and $\lambda_b$, $\sigma_b$ fixed at -80, yields errors that are not significantly higher than those of the eight variable fits. As shown from the five variable and the 2 variable fit, fixing the amplitude-scaling factor at 16, average amplitude from the eight variable fits, yields significantly higher errors for Churchill, 1994-1995 and Arviat, 1997-1998. Thus as a set, the three variable fits appear to be significantly better than the two
variable fits. Although it is uncertain why a single value $s_{amp}$ could not be found that will render excellent fits for all of the intervals, the value of 16 for $s_{amp}$ does render relatively good fits for the other four intervals. The results of this three variable fitting are shown in Figure 5.8, where again the solid line represents the data, the dotted line represents the standard FB model and the dashed line represents the best fits in the case where $s_{amp}$, $T_{max}$, and $T_{width}$ were allowed to vary in the fittings.

The best fit is achieved for

$$\lambda_a = \sigma_a \approx 28$$  \hspace{1cm} (5.10)

$$\lambda_b = \sigma_b \approx -80$$  \hspace{1cm} (5.11)

which results in the relationship:

$$R \approx 28K_p - 80$$  \hspace{1cm} (5.12)

Also $C_{p\beta}$ was determined to have a best fit value of 0.15. However it should be noted that small changes to this variable does not have as significant an effect as the adjustment of the other variables.

Substituting Equation 5.12 back into the Foppiano and Bradley model yields modified expressions for the $K_p$ dependence in the model.

$$\lambda'_m = 70.2(1 - .016K_p) + .012(1 + 8.4K_p)|\epsilon|$$  \hspace{1cm} (5.13)

$$\sigma_\lambda = 2.04(1 + .16K_p)$$  \hspace{1cm} (5.14)
Table 5.11: Plots of the best fit found (dashed curve) by varying only $s_{amp}$, $T_{max}$ and $T_{width}$ of the Kp model. The standard non-Kp results of the Foppiano and Bradley model (dotted curve) are presented as a baseline and the data is the solid line.
5.6.3 Fitting the Alternate Kp Dependent Model

The alternate model for the Kp dependence, which also replaces $s_R$ with $s_{Kp}$ was tested in the same way as the previous model. The results of fitting this model to our data are summarized in Table 5.12.

Comparing the errors of the alternative model shows no significant change in error values, and therefore yields comparable fits, as compared with the Kp dependent model presented in the previous section. Although the fits between the two models are comparable, the values of the Kp dependent variables are significantly different as follows:

\[
\lambda_a = \sigma_a \approx 33 \tag{5.15}
\]

\[
\lambda_b = \sigma_b \approx -65 \tag{5.16}
\]

resulting in the relationship:

\[
R \approx 33Kp - 65 \tag{5.17}
\]

Substituting Equation 5.17 back into the Fopiano and Bradley model yields:

\[
\lambda'_{m} = 69.6(1 - .019Kp) + .066(1 + 1.8Kp)|\tau| \tag{5.18}
\]

\[
\sigma_{\lambda} = 2.22(1 + .178Kp) \tag{5.19}
\]

\[
s_{Kp} = .42(1 + .72Kp) \tag{5.20}
\]
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Table 5.12: Table of all the fitting runs for the alternate model. Bold-face variable values are fitted and other variable values are fixed.
Equation 5.17 is much closer to Foppiano and Bradley's mean relationship:

\[ R = 33.1Kp - 51.6 \]  \hspace{1cm} (5.21)

and the value of $C_{joE}$ is slightly lower at 0.1.

However, since both models of these modified versions of the Foppiano and Bradley model produce very similar fits, it is unclear which modified model is better, and more research and data analysis would be needed in comparing the two models. Both do appear distinctly superior to the original FB model for explaining our data.
Table 5.13: Plots of the best fit found (dashed curve) by varying only $s_{emp}$, $T_{max}$, and $T_{width}$ using the alternate Kp model. The standard non-Kp results of the Foppiano and Bradley model (dotted curve) are presented as a baseline and the data is the solid line.
Chapter 6

Concluding Remarks

Previous studies have modeled the absorption effects of the aurora by using a known propagation path from a known transmitter to a known receiver. However, we have shown that it may be feasible to test models of auroral absorption with an experiment involving large numbers of transmitters even when the transmitters and propagation paths received are not exactly known. This was done by estimating the placements of the transmitters and simulating the major aspects of path loss for each of the simulated propagation paths. By resolving the different propagation paths, and estimating the net transmitted signal strength on each path, we were able to simulate the levels of absorption in the aggregate signals received at the Canadian sites.

By comparing the results of the simulation to the data, we were able to determine both qualitatively and quantitatively that the auroral absorption in the data generally followed the Foppiano and Bradley model. However, by comparing data from multiple years we were also able to determine that the index of geomagnetic activity, Kp, is an important parameter and should be included in the model. Two different possible methods of the way in which the model accounts for Kp were presented. When used with our data set, both of these augmented models generate comparable results. More research would need to be done to determine which method better modeled auroral
absorption.

By setting up a search algorithm to run the simulation multiple times and compare the result quantitatively to the data using the standard error estimate, we were able to fit several variables in the model to the data. The hill-climbing algorithm used by the search to minimize the error proved to be an effective means of searching for best fits, even when there were eight variables to be fitted. Although the search may not have found the absolute best fit, it found good fits in the neighborhood of the Foppiano and Bradley parameters, which are in turn based on a large independent data set and are believed to be a good model. It generally found a fit that in all likelihood was not significantly worse than the absolute best fit. Most importantly the search would find a reasonable fit in a matter of minutes for the eight variable fit, while a comprehensive search would in theory take over two years to complete.

This thesis offers some new insights into modeling auroral absorption. However, more work is still needed to perfect the model. The most important next step to take is to assess how the placement of the virtual transmitters in the simulation affects the results. Although an evenly spaced grid of transmitters was used in this thesis, placing transmitters on major population centers, or clustered on the east coast may be a better approximation of the transmitter placement and could affect the results. Also greater experimental and theoretical research is needed to asses the differences between the two Kp models presented to see if one of the models is more valid than the other. Also more research can be devoted to developing and analyzing other possible Kp models that were not presented in this thesis.

Although this thesis concentrated on the lower latitude regions of the aurora, data from higher latitude regions may yield a more complete picture. Furthermore we were only able to analyze data from a portion of the solar cycle. Analyzing data from an entire solar cycle especially from a solar maximum, which was not covered,
would yield a better understanding of how important the solar epoch variable is to the model. Also analyzing more data would help in refining the Kp dependent model. Lastly, although this thesis concentrated only on late fall and winter data, it would be potentially interesting to analyze data from all months in attempts to better determine the seasonal dependence of the aurora.

The simulation presented in Chapters 2 and 3 appears to be a feasible means of testing models of auroral absorption, and the data fitting algorithm presented in Chapter 5 seems to be an efficient aid in expanding the models. One advantage to using the method described is that it does not require a transmitter in the experimental apparatus, since the method exploits the reception of signals from a very large number of sources which are operating independently. The feasibility of the simulation can be seen since it did a reasonable job of predicting the splash absorption in our data using the standard Foppiano and Bradley model, and we were able to improve the fits by adjusting the model. Thus it seems possible to create and test models of auroral absorption, using only receivers, and simulating the transmitters.
Appendix A

The Drizzle Terms of the Foppiano and Bradley Model

Like the model for the splash precipitation, Foppiano and Bradley [1983] base drizzle precipitation upon corrected geomagnetic latitude (\(\lambda\)), magnetic local time (T), solar epoch (R), corrected geomagnetic longitude (\(\theta\)), and season (M). \(Q_{1d}\), the percentage of days that the absorption due to drizzle precipitation measured by a riometer at 30 MHz is given by:

\[
Q_{1d} = 21d_\lambda d_\theta d_R d_q d_m
\]  
(A.1)

The latitude dependence in the FB model is:

\[
d_\lambda = e^{\exp[-\frac{(\lambda - \lambda_m)^2}{2\sigma_\lambda^2}]}
\]  
(A.2)

where the peak latitude of the drizzle in degrees is:

\[
\lambda_m = 68(1 - 0.0004R)
\]  
(A.3)
and the width of the latitude Gaussian in degrees is:

\[ \sigma_\lambda = 3(1 + 0.004R) \] (A.4)

\( \lambda \) is the corrected geomagnetic amplitude in degrees.

The diurnal dependence of the drizzle is:

\[ d_T = \exp\left(\frac{-(t_1 - T_m)^2}{15.7}\right) \] (A.5)

where

\[ t_1 = \begin{cases} 
T & \text{for } 0 \leq T \leq T_m + 12 \\
T - 24 & \text{for } T_m + 12 < T < 24 
\end{cases} \] (A.6)

where \( T \) is the corrected geomagnetic local time in hours.

The solar epoch dependence is:

\[ d_r = 1 + 0.14R \] (A.7)

where \( R \) is the smoothed sunspot number.

The longitude dependence of the drizzle is the same as the longitude dependence of the splash, which is

\[ d_\theta = \begin{cases} 
0.58 - 0.42\sin[0.947(\theta + 85^\circ)] & \text{for } 0 \leq \theta < 10 \\
0.16 & \text{for } 10 \leq \theta < 80 \\
0.58 + 0.42\sin[1.80(\theta + 130^\circ)] & \text{for } 80 \leq \theta < 130 \\
0.58 - 0.42\sin[0.947(\theta - 275^\circ)] & \text{for } 180 \leq \theta < 360 
\end{cases} \]
where $\theta$ is the corrected geomagnetic longitude in degrees.

Unlike the splash the drizzle precipitation does have a seasonal dependence in the Foppiano and Bradley model, which is given for the northern hemisphere as:

$$d_m = 1 - 0.3 \sin 3.86\delta$$  \hspace{1cm} (A.9)

where $\delta$ is the mean monthly solar declination in degrees.

Foppiano and Bradley extend the model to include a $Kp$ dependence for the splash by replacing Equations A.3 and A.4 with

$$\lambda_m = 68.9(1 - 0.014Kp)$$  \hspace{1cm} (A.10)

and

$$\sigma_\lambda = 2.16(1 - 0.17Kp)$$  \hspace{1cm} (A.11)
Appendix B

Code to Calculate the $\Phi$ Function

The function $\Phi(f_{\text{los}})$ illustrated in Figure 2.3 is critical in calculating the absorption of radio waves at oblique incidence. The code to generate the $\Phi$ function from George and Bradley [1974] is given below courtesy of P.A. Bradley.

```c
double Phi(double fv, double foe)
{
    double x, f_ratio;
    double value;

    /* at extreme foe = 0 phi goes to 1 */
    if(foe == 0.0)
        return 1.0;
    else
        f_ratio = fv/foe;

    /* Function given in George and Bradley 1974 */
    if (f_ratio >=0.0 && f_ratio < 1.0)
    {
        x = (f_ratio -.475)/.475;
        value = .225 + (.159*x) + .044*(pow(x,2.0)) - (.027*pow(x,3.0))
            +(.127*pow(x,4.0)) +(.04*pow(x,5.0)) -(.093*pow(x,6.0));
        if (value > .53)
            value = .53;
    }
```

else if (f_ratio >= 1 && f_ratio <= 2.2)
{
    x = (f_ratio-1.65)/.55;
    value = .375 - .049*x + .054*\text{pow}(x, 2.0)
    + .034*\text{pow}(x, 3.0) -.027*\text{pow}(x, 4.0) -.07*\text{pow}(x, 5.0) + .043*\text{pow}(x, 6.0);
    if (value > .53)
        value = .53;
}
else if (f_ratio > 2.2 && f_ratio <= 10)
{
    value = .34 + (0.02/7.8)*(10.0 - f_ratio);
}
else if (f_ratio > 10)
{
    value = .34;
}
return value/.34;
Appendix C

Software Reference

As shown in Figure C.1, the software is comprised of three parts: the simulation, the data processing unit, and the intelligent search module.

![Diagram of software design](image)

**Figure C.1:** The overall design of the software. Ovals represent input/output and the rectangles represent software execution.
C.1 The Simulation

The simulation module, shown in plain font in Figure C.1, can be run independently from the rest of the software. There are three executables which produce plots of different aspects of the simulation. The program ‘simpplot’ plots $Q_1$ vs. UT for Kp values of one through five and the program ‘obliqueplot’ plots total path loss vs. UT. Both ‘simpplot’ and ‘obliqueplot’ assume a system containing only a single specified transmitter and receiver. The program ‘multitransmit.plot’ works like ‘obliqueplot’ except that it assumes a system containing multiple transmitters as specified in the file ‘transmitter.dat’. The executable ‘multitransmit.plot’ runs both the Kp dependent and Kp independent models. In addition the simulation module utilizes an extensive set of library functions for calculating propagation paths, and ionospheric absorption, and net path loss.

C.2 The Data Processing Unit

The data processing unit, shown in italics, is responsible for reading the raw CANOPUS data and outputting the average signal strength for a given interval of days. It is comprised of two programs: ‘extract_avg’ and ‘plot_data’.

The program ‘extract_avg’ takes a raw CANOPUS datafile as input and returns the integrated power for that day for a specified frequency range in five minute intervals. The output files were then archived to be used by plot_data.

The program ‘plot_data’ reads a specified set of output files from ‘extract_avg’ and outputs the average signal strength in decibels for the range of days for a specified frequency and polarization as described in its man page. It can return either a plot or a datafile of average signal strength vs. UT as specified by the user. The user may also specify whether the output is to be sorted by Kp index.
C.3 The Search Module

The goodness of fit heuristic and the intelligent search algorithm comprises the search module, shown in bold in Figure C.1. The goodness of fit function uses the outputs from both the simulation and the data processing unit to calculate the least squares error between the two outputs. The intelligent search algorithm drives the simulation, and implements a hill-climbing algorithm to look for the set of variable values that will minimize the error between the simulation’s output and the data. The program benchmark_plot incorporates both the heuristic function and the search function.

C.4 Manual Pages

The following manual page summarizes describe the following programs: simplot, obliqueplot, multitransmit_plot, extract_avg, plot_data, and benchmark_plot utilized in the software design.
NAME

simplot – produces a Q1 vs UT graph using the Foppiano and Bradley model

SYNOPSIS

simplot [-Pprinter] <S1 lat> <S1 long> <R> <day> <reflect ht>

DESCRIPTION

This program plots Q1 vs UT using the Foppiano and Bradley model. The plot is saved as a postscript file called “simplot.ps”. The program takes as arguments the geographical latitude and longitude of the transmitter and receiver as well as the sun spot number R, the day number, and the virtual reflection height of the signal (km). The plot is computed by adding the Q1’s at the point of entry and exit of the D region for a 30MHz radio wave at vertical incidence on a 1 hop path. The output consists of 5 graphs corresponding to kp = 1 to 5.

-Pprinter: This option will also print the postscript file to the specified printer. If no printer is specified the output will be sent to the default printer.

SEE ALSO

oblique_plot multitransmit_plot plot_data

REVISIONS

Sun Aug 29 12:30:14 EDT 1999
NAME
obliqueplot – produces an absorption vs UT graph for waves at oblique incidence

SYNOPSIS
obliqueplot [-Pprinter] [-Smin_max] [-r] <S1 lat> <S2 lat> <S2 long> <R> <day>

DESCRIPTION
This program plots Path Loss in decibels vs UT using the Foppiano and Bradley model. The
plot is saved as a postscript file called "simplot.ps". The program takes as arguments the geographical
latitude and longitude of a transmitter and reciever as well as the sun spot number R, the day number,
observerd frequency (Mhz) the virtual reflection height of the signal in kilometers and a foe factor. The
foe is computed by multiplying the factor by the value of Q1. The plot is computed by resolving the
absorptions along different paths of up to 3 hops using the inverse sum formula. For each path the loss
is the sum of the oblique absorption at the points of entry into the D region added to any ground
reflection loss (4db/reflection) added to the loss due to path attenuation. The ionospheric absorption is
the sum of the auroral absorption and the typical daytime absorption.

-Pprinter: This option will also print the postscript file to the specified printer. If no printer is
specified the output will be sent to the default printer.

-S[min_max]: Scaling option for the y axis. If this is not specified autoscale will be used.
Min is bottom value of the y axis and max is the top. If max is less than min the graph will be plotted
upside down.

-r: This option plots the different kp graphs relative to kp = 1.

SEE ALSO
simplot multitransmit_plot benchmark_plot plot_data extract_avg

REVISIONS
Sun Aug 29 12:30:14 EDT 1999

Figure C.3: Man page of obliqueplot
NAME
multitransmit_plot – produces a Path Loss vs UT graph for waves at oblique incidence by resolving signals from multiple transmitters.

SYNOPSIS
multitransmit_plot [-Pprinter] [-Smin_max] [-R | -r] <R> <day> <fob> <reflect_ht> <foe_fact>

DESCRIPTION
This program produces a graph of path loss vs UT in a similar manner as obliqueplot except that it takes into account multiple transmitters. The signals from the multiple transmitters are resolved using the inverse sum formula.

-Pprinter: This option will also print the postscript file to the specified printer. If no printer is specified the output will be sent to the default printer.

-S[min_max]: Scaling option for the y axis. If this is not specified autoscale will be used. Min is bottom value of the y axis and max is the top. If max is less than min the graph will be plotted upside down.

-R: Runs simulation without sorting by Kp. Cancels any previous -r options.

-r: This option plots the different kp graphs relative to kp = 1. Cancels any previous -R options.

FILES
This program must be able to see the file "transmitter.dat" which contains a list of geographical coordinates of transmitters. The file should contain 2 columns, the first being geographical latitude and the second being geographical longitude. Each line represents a different transmitter.

SEE ALSO
simplot obliqueplot benchmark_plot plot_data extract_avg

REVISIONS
Sun Aug 29 12:30:14 EDT 1999
Appendix C: Software Reference

NAME
   extract_avg – Creates .avg files from CANOPUS data files

SYNOPSIS
   extract_avg [-a] [-r] [-Dfile_path] [-Ooutfile_dir]

DESCRIPTION
   extract_avg reads files from a CANOPUS radio data CDROM and outputs .avg files that can be
   used plot_data to plot signal strength and absorption for different kp’s over a long period of time. The
   filenames have the form “ssYYddd-rr.avg”. The output files contain a timestamp (20 - 40 UT) a kp
   value and intensities for 6 frequencies: 550-1000 L/R, 1300-1600 L/R & 3600-4400 L/R.
   extract_avg also copies the needed kp_xx.dat files from /usr/local/bin/radio_3 to current directory which can be removed with the -r option.

   -a: Outputs the averages the left and right polarizations for each frequency. It still does output
   the same number of columns though some of the columns will now contain the same data.

   -r: Removes kp_xx.dat files from current directory.

   -Dpath: Specifies the location of the CANOPUS radio files for the program to average. The
   default is CD2 on newton.

   -Opath: Specifies the directory where all the output files should be placed. The default is
   /usr/radio/data9/eriedata/

FILES
   Files to be read must be CANOPUS radio files.
   Must also be able to read appropriate kp_xx.dat files.
   Runs integrate & int2avg.f

SEE ALSO
   plot_data

REVISIONS
   Fri Jan 21 01:24:54 EST 2000
NAME
plot_data – Creates postscript plots of average signal strength of absorption for different values of kp.

SYNOPSIS

DESCRIPTION
plot_data looks for .avg files created by extract_avg and plots them using milan_analysis and
milan_histo. The output is a postscript file with the name in the form "ss(YY)d_ddd_ddd.ps". It can plot
signal strength for each value of kp or absorption relative to kp = 1 if -r option is specified.

-Pprinter: Sends the output directly to the specified printer. If only -P is specified, the output
goes to the default printer.

-S[min_max]: Scaling option for the y axis. If this is not specified autoscale will be used.
Min is bottom value of the y axis and max is the top. If max is less than min the graph will be plotted
upside down.

-R: Plots data without sorting by Kp. Cancels any previous -r or -c options
-r: Creates absorption plots relative to kp = 1 (db). If this is not specified the plots will be
average signal strength (db). Cancels any previous -R options.

-c: Combine kp = 4 and kp = 5 into one plot (kp = 4). Cancels any previous -R options.

-Ssite: Specifies which site to plot with the default being Churchill (ch). Other legal sites
include Arviat (ar), Baker Lake (bl), Gillam (gi) and Taloyoak (ta). The 2 character code should be
specified with this option.

-Yyear: Specify the year in 2 digit form. If not specified the program will include files from all
the years it can find.

-dBEGINday_ENDday: Specify the day range of files to include in the plotting. Default is 1-366.

-Dpath: Specify the location of the .avg files to analyze. The default is the current directory.

-Mmode: Specify the frequency and polarization to analyze:
1 = 550-1000 Left
2 = 550-1000 Right
3 = 1300-1600 Left
4 = 1300-1600 Right
5 = 3600-4500 Left
6 = 3600-4500 Right
7 = Average of 3600-4500

-F: Outputs a datafile instead of a postscript plot

FILES
Files to be read must be .avg files as created by extract_avg.
Runs milan_analysis, milan_histo, combine_kp, f rel_data, c time_adj, c

SEE ALSO
extract_avg

Figure C.6: Man page of plot_data
NAME
benchmark_plot – Outputs a plot that attempts to find the best fit between the model and the data. Includes the nonKp Foppiano and Bradley plot as a baseline reference.

SYNOPSIS
benchmark_plot [-Pprinter] [-Smin_max] <lat> <long> <R> <Kp> <day> <fob> <reflect_ht> <datafile>

DESCRIPTION
This program runs the hill-climbing algorithm to attempt to find the best fit between the variables of the simulation and the data. The program quantifies the goodness of fit using the least squares error. The program accepts datafiles produced using "plot_data" with both the -R and -F options.

The program can fit both the nonKp dependent and the Kp dependent models. To fit the nonKp dependent model for the data, enter zero as the argument for Kp. To fit Kp model enter a value greater than .1 for Kp.

The following variables are fit using the program:
S_amp, S_tmax, T_width, C_foe, Lbd_a, Lbd_b, Sig_a, Sig_b

-Pprinter: This option will also print the postscript file to the specified printer. If no printer is specified the output will be sent to the default printer.

-Smin_max: Scaling option for the y axis. If this is not specified autoscale will be used. Min is bottom value of the y axis and max is the top. If max is less than min the graph will be plotted upside down.

FILES
The input datafile should have been produced by plot_data -R -F ...

Outputs a postscript plot named "simplot.ps" and a textfile named "logbook.txt" which contains the search history, the values of the variables of the best fit and the errors for the best fit and the Foppiano and Bradley curve.

SEE ALSO
simplot obliqueplot multitransmit_plot plot_data extract_avg

REVISIONS
Sun May 7 17:10:01 EDT 2000
References


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