

General Instructions: Same as in Homework 1.

Honor Principle: For this homework, you should work entirely on your own and not discuss with anyone.

14. Give a full formal proof that $ZPP = RP \cap \text{coRP}$. [2 points]

15. For constants $0 < \alpha < \beta < 1$, define the class $\text{BPP}_{\alpha,\beta}$ to be the class of all languages $L \subseteq \Sigma^*$ such that there exists a PTM M that runs in polynomial time and behaves as follows on an input $x \in \Sigma^*$:

$$\begin{aligned}x \notin L &\Rightarrow \Pr[M \text{ accepts } x] \leq \alpha, \\x \in L &\Rightarrow \Pr[M \text{ accepts } x] \geq \beta.\end{aligned}$$

Note that our definition of BPP in class coincides with $\text{BPP}_{\frac{1}{3}, \frac{2}{3}}$ in this notation.

Using Chernoff bounds, give a full formal proof that for all α and β as above, $\text{BPP}_{\alpha,\beta} = \text{BPP}$.

[2 points]