General Instructions: Same as in Homework 1.

Honor Principle: Same as in Homework 1.

For this homework, we use the notation PCP_{c,s}[r,q] for the class of languages that have probabilistically checkable proof systems with completeness c, soundness s, using r random bits, and making q queries. All of these parameters can be functions of n, the input length.

When c and s are unspecified, the default values are c = 1 and s = \frac{1}{2}. We also define the shorthand notation PCP[\log, \text{const}] = \bigcup_{c,d=1}^{\infty} \text{PCP}[c \log n, d]. The PCP theorem then states that NP = \text{PCP}[\log, \text{const}].

27. Recall that the querying done by the verifier in a PCP system is required to be non-adaptive.

Let A be a language. Prove that if A has a PCP-verifier using r random bits and q adaptive queries (i.e., the locations to be queried may depend on the outcomes of previous queries), then A \in \text{PCP}[r, 2^q].

28. Prove that if SAT \in \text{PCP}[r(n), \text{const}] for some function r(n) = o(\log n), then P = NP.