Homework 4 (Grader: Huang)
Due Feb 8, 2006 by 10:00am

Prof. Amit Chakrabarti
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Please submit your solutions into the box marked "CS 19 Homework In" at the main entrance of Sudikoff.

You must demonstrate how you arrived at your final answers — i.e., you must show your steps — unless the problem statement makes an exception. You must also justify any steps that are not trivial. Simply writing down a final answer *will not earn any credit*. Please think carefully about how you are going to organise your answers *before* you begin writing.

The notation P$i.j$-k refers to Problem k from the list of problems after Section $i.j$ in your textbook. Thus, P1.2-4 refers to Problem 4 on page 17.

1. Solve P4.1-4, using mathematical induction. [10 points]

2. Solve P4.1-6, giving two proofs: a proof via mathematical induction on n, *as well as* a combinatorial proof. The textbook calls the latter a "story" proof.

   Hint for the combinatorial proof: partition the set of all $(j+1)$-element subsets of $\{1, 2, \ldots, n+1\}$ based on the largest integer in the subset. [20 points]

3. Solve P4.2-2. It will take some effort to get the algebra right, so please work it out fully before you write the final version of your solution. Remember that a proof must be written to show the logical flow of ideas. You might have worked out your proof by starting from what you want and simplifying down, but *your proof should start from what you know and get to what you want*. Proofs not written this way will not get much credit. [15 points]

4. Solve P4.2-8. [10 points]

5. Solve P4.2-14. [10 points]

6. Solve P4.2-17. [15 points]

7. Solve P4.3-3. Please draw out your recursion tree neatly, following the same scheme as the examples in the book. [10 points]

8. Solve P4.3-5. The same comment as above applies. [10 points]

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Challenge Problems

These challenge problems are intended to provide a higher level of challenge for those who want to think further about discrete mathematics. They are not for regular credit and not solving them will not hurt you in any way. If you do solve one of these, please turn in your solutions **directly to the instructor**.

CP1: Let p, m and n be positive integers and suppose p is prime. Prove that $\binom{pm}{pn} - \binom{m}{n}$ is divisible by p^2.

This problem can be solved by staring at a big Pascal's triangle and thinking very hard!

CP2: Solve P4.2-18. That's the warm-up. Now here's the real problem: give a general method to solve a recurrence of the form

$$T(n) \;=\; \begin{cases} aT(n-1) + bT(n-2), & \text{if } n \geq 2\,, \\ c, & \text{if } n = 1\,, \\ d, & \text{if } n = 0\,, \end{cases}$$

where a, b, c and d are positive real numbers.

If you attempt this problem, your first step should be to ask yourself how the constants $(1+\sqrt{5}/2)$ and $(1-\sqrt{5}/2)$ appeared in P4.2-18.