Homework 5 (Grader: Blinn)
Due Feb 15, 2006 by 10:00am

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Please submit your solutions into the box marked "CS 19 Homework In" at the main entrance of Sudikoff.

You must demonstrate how you arrived at your final answers — i.e., you must show your steps — unless the problem statement makes an exception. You must also justify any steps that are not trivial. Simply writing down a final answer *will not earn any credit*. Please think carefully about how you are going to organise your answers *before* you begin writing.

The notation P$i.j$-k refers to Problem k from the list of problems after Section $i.j$ in your textbook. Thus, P1.2-4 refers to Problem 4 on page 17.

1. Solve P4.3-14, parts (b) and (c). Please draw out your recursion tree neatly, following the same scheme as the examples in the book. [15 points]

2. Solve P4.4-1 (all five parts). Do not draw recursion trees; just use the master theorem. You must show the steps that led you to one of the three cases of the master theorem. [15 points]

3. Solve P4.4-2, P4.4-3, P4.4-4 and P4.4-5. Use the general version of the master theorem that handles ceilings. [20 points]

4. Solve P4.5-3. Mimic the style of the induction proofs in exercises 4.5-{1,2,3}. [15 points]

5. Problem P4.5-5 asks you whether the $O(n \log_3 n)$ bound in the previous problem is in fact a $\Theta(n \log_3 n)$ bound. Solve this problem and *explain your answer*; i.e., if you think that the answer is "no," then state and prove the correct big-Θ bound. Note that the master theorem does not apply in this case, so you will not be able to use it. [15 points]

6. Let α, β and c be positive real constants with $\alpha + \beta < 1$. Suppose $T(n)$ is a sequence defined on the integers that satisfies the inequality
$$T(n) \;\leq\; T(\lceil \alpha n \rceil) + T(\lceil \beta n \rceil) + cn\,.$$
Give a careful proof, using induction and the precise definition of big-O, that $T(n) = O(n)$. [20 points]

Challenge Problems

These challenge problems are intended to provide a higher level of challenge for those who want to think further about discrete mathematics. They are not for regular credit and not solving them will not hurt you in any way. If you do solve one of these, please turn in your solutions **directly to the instructor**.

CP3: Prove by mathematical induction that for all integers $n \geq 1$,
$$\binom{2n}{n} \;<\; \frac{4^n}{\sqrt{3n}}\,.$$
If you try to solve this mechanically you'll find that your proof fails. This is another example of a situation where your inductive hypothesis was not strong enough to prove the statement (compare this with Exercise 4.5-3 from the textbook). Find an appropriate stronger statement and prove that by induction.