Homework 6 (Grader: Blinn)
Due Feb 22, 2006 by 10:00am

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Please submit your solutions into the box marked "CS 19 Homework In" at the main entrance of Sudikoff.

You must demonstrate how you arrived at your final answers — i.e., you must show your steps — unless the problem statement makes an exception. You must also justify any steps that are not trivial. Simply writing down a final answer *will not earn any credit*. Please think carefully about how you are going to organise your answers *before* you begin writing.

The notation P$i.j$-k refers to Problem k from the list of problems after Section $i.j$ in your textbook. Thus, P1.2-4 refers to Problem 4 on page 17.

1. Solve both parts of P5.1-6. [10 points]

2. Solve P5.1-11. [5 points]

3. Poker is a rather mathematical card game with many variations, but all based on the basic principle that certain hands "beat" (i.e., have higher value than) certain others. A hand consists of five distinct cards chosen from a standard deck of 52 cards. There are nine special hands and they are given specific names, as follows.

   A *royal flush* consists of the Ace, King, Queen, Jack and 10, all of the same suit. A *straight flush* is any five-card sequence within a suit, except for the one beginning with the Ace (that would make it royal), for instance, Jack, 10, 9, 8, 7. A *straight* is any five-card sequence with not all cards of the same suit and a *flush* is a set of five cards of the same suit but not in sequence.

   A *four-of-a-kind* is hand with four cards of the same value (e.g. with four 9s). A *three-of-a-kind* has three cards of the same value and two other cards with two other different values; had these two other cards had the same value that would give us a *full house*; thus, three 5s, a King and a 9 give us a three-of-a-kind whereas three 5s and two 10s give us a full house. A *two pair* contains two different equal-value pairs and an unrelated fifth card, e.g. two 7s, two Kings, and an Ace. Finally, a *pair* has just one equal-value pair and three other unrelated cards.

   (a) Suppose you shuffle a deck of 52 cards and then draw five cards at random. What is the probability of getting each of the nine special hands (royal flush, straight flush, straight, flush, four-of-a-kind, three-of-a-kind, full house, two pair and pair)?

   (b) A sensible design of the rules of poker would ensure that if you have been lucky enough to draw an "unlikely" hand, then your hand should beat that of an opponent who has drawn a more "mundane" hand. In fact, poker *was* sensibly designed. By arranging the hands from least likely to most likely, figure out the "pecking order" of these special hands in poker. You might want to use a calculator unless you're a whiz with numbers!
   [20 points]

4. Solve P5.2-10. You will want to use the principle of inclusion and exclusion and your final answer will be a formula using summation (i.e., Σ) notation. It is not possible to simplify the summation, so leave it at that.
   [10 points]

5. Find the number of integers between 1 and $10{,}000$ (inclusive) that are not divisible by 4, nor by 5, nor by 6.
   [10 points]

6. Solve P5.3-2. [10 points]

7. Solve P5.3-7. [10 points]

8. Solve P5.3-8. [10 points]

9. This problem should convince you not to trust vaguely formed "intuitions" about probability, but instead to carefully work out the numbers using the proper definitions and theorems from probability theory. It is the famous (some would say infamous) Monty Hall problem, which gets its name from the TV game show *Let's Make A Deal*, hosted by Monty Hall.

   You are asked to select one closed door of three, behind one of which there is a prize. The other two doors hide nothing. Once you have made your selection, Monty Hall opens one of the remaining doors, revealing that it does not contain the prize. He then asks you if you would like to switch your selection to the other unopened door, or stay with your original choice. The problem: should you switch?

   Work this out meticulously. Carefully define a sample space, define any necessary events and then work out two probabilities:

   (a) The probability that you win the prize if you don't switch.

   (b) The probability that you win the prize if you do switch.

   Do you find the answers intuitive? (There is no incorrect answer to that question!) If not, the lesson you have learnt is that you need to wait until your intuition has matured before trusting it. [15 points]