Homework 7 (Grader: Huang)
Due Mar 1, 2006 by 10:00am

Prof. Amit Chakrabarti
Computer Science Department
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Please submit your solutions into the box marked "CS 19 Homework In" at the main entrance of Sudikoff.

You must demonstrate how you arrived at your final answers — i.e., you must show your steps — unless the problem statement makes an exception. You must also justify any steps that are not trivial. Simply writing down a final answer *will not earn any credit*. Please think carefully about how you are going to organise your answers *before* you begin writing.

The notation P$i.j$-k refers to Problem k from the list of problems after Section $i.j$ in your textbook. Thus, P1.2-4 refers to Problem 4 on page 17.

1. Solve P5.4-6. [5 points]

2. Solve P5.4-12. [10 points]

3. Consider the geometric random variable X in Theorem 5.13, i.e., X is the number of trials until the first success in a Bernoulli trials process with probability p of success. Assume $0 < p < 1$. Let m and n be positive integers. Evaluate $P(X \geq n \mid X \geq m)$. Make sure you handle all possible cases! [10 points]

4. Solve P5.5-2. [20 points]

5. Solve P5.5-4. [5 points]

6. Solve P5.5-14. [10 points]

7. Solve P5.6-3. [10 points]

8. Solve P5.6-11. [15 points]

9. Prove Theorem 5.23. The intuition you gained by doing this proof should now help you prove a famous theorem about conditional probabilities called Bayes' Theorem, which states the following. Let S be a sample space for a random experiment and let $A_1, \ldots, A_n \subseteq S$ be mutually exclusive events whose union is S. Then, for any event $B \subseteq S$,
$$\forall i \quad P(A_i|B) \;=\; \frac{P(A_i)P(B|A_i)}{\sum_{k=1}^{n} P(A_k)P(B|A_k)}\,.$$
Give a rigorous proof of Bayes' Theorem. [15 points]

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Challenge Problems

These challenge problems are intended to provide a higher level of challenge for those who want to think further about discrete mathematics. They are not for regular credit and not solving them will not hurt you in any way. If you do solve one of these, please turn in your solutions **directly to the instructor**.

CP4: During a particularly boring one-hour lecture, each of Alice, Bob and Carl fell asleep for a random (but contiguous) 20-minute interval. The three sleep intervals were independent of each other and uniformly distributed. What is the probability that there was a point of time during the lecture when all three of Alice, Bob and Carl were asleep?

Note: As stated, this problem deals with *continuous* sample spaces and probability distributions. But it *can* be solved using only what you know if you make some suitable extra assumptions. A fully rigorous solution requires calculus (but not beyond the level of Math 8).