Homework 8 (Grader: Huang)
Due Mar 8, 2006 by 10:00am

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Please submit your solutions into the box marked "CS 19 Homework In" at the main entrance of Sudikoff.

You must justify any step that is not trivial. Please think carefully about how you are going to organise your answers *before* you begin writing. The notation P$i.j$-k refers to Problem k from the list of problems after Section $i.j$ in your textbook. Thus, P1.2-4 refers to Problem 4 on page 17.

All graphs are assumed to be *simple*, i.e., no loops and no instances of multiple edges between the same pair of vertices.

1. Solve P6.1-9. [10 points]

2. Solve P6.1-13. Of course, you must prove your answer to earn credit. [15 points]

3. Solve P6.1-14. [15 points]

4. Solve P6.1-15. [15 points]

5. Let G be an n-vertex graph where the degree of the i^{th} vertex is d_i. Prove that for all k with $1 \le k < n$ we have the following inequality:
$$\sum_{i=1}^{k} d_i \;\le\; k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}\,.$$
[15 points]

6. Solve P6.3-2. [10 points]

7. Solve P6.3-4. [10 points]

8. Solve P6.3-6 and give a proof justifying your answer. [10 points]

Challenge Problems

CP5: Let G be a graph in which every vertex has degree either k or $k+1$ (for some integer k). Prove that G has a subgraph in which every vertex has degree either k or $k-1$.

Note: The solution involves concepts from Section 6.4, which we shall cover during the final week of classes.