CS 19: Discrete Mathematics

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Logic and Logical Notation

Proposition

A declarative sentence that is either true or false, but not both. Examples:

- CS19 is a required course for the CS major.
- Tolkien wrote *The Lord of the Rings*.
- Pigs can fly.

Non-examples:

- What a beautiful evening!
- Do your homework.

Compound Proposition

One that can be broken down into more primitive propositions. E.g.,

- If it is sunny outside then I walk to work; otherwise I drive, and if it is raining then I carry my umbrella.

This consists of several primitive propositions:

p = "It is sunny outside" q = "I walk to work"
r = "I drive" s = "It is raining"
t = "I carry my umbrella"

Connectives: "if… then", "otherwise", "and"

Compound Proposition

If it is sunny outside then I walk to work; otherwise I drive, and if it is raining then I carry my umbrella.

p = "It is sunny outside" q = "I walk to work"
r = "I drive" s = "It is raining"
t = "I carry my umbrella"

If p then q; otherwise r and if s then t.

If p then q and (if not p then (r and (if s then t))).

p implies q and ((not p) implies (r and (s implies t))).

Logical Connectives

Used to form compound propositions from primitive ones.

English name	Math name	Symbol	Java operator
"and"	conjuction	$\wedge$	&&
"or"	disjunction	$\vee$	||
"not"	negation	$\neg$	!
"or… but not both"	xor	$\oplus$	-none-
"if…then"	implication	$\Rightarrow$	-none-
"if & only if"	equivalence	$\Leftrightarrow$	-none-

Compound Proposition, in Symbols

If it is sunny outside then I walk to work; otherwise I drive, and if it is raining then I carry my umbrella.

p = "It is sunny outside" q = "I walk to work"
r = "I drive" s = "It is raining"
t = "I carry my umbrella"

p implies q and ((not p) implies (r and (s implies t))).

$$(p \Rightarrow q) \wedge (\neg p \Rightarrow (r \wedge (s \Rightarrow t)))$$

Defining a Logical Connective

Consider a connective that combines two propositions, e.g. implication "$p \Rightarrow q$". There are exactly four possibilities:

- p is true, q is true
- p is true, q is false
- p is false, q is true
- p is false, q is false

In each case, specify the truth value of "$p \Rightarrow q$".

Pondering Implication

s = "It is raining", t = "I carry my umbrella", $s \Rightarrow t$

Think of the implication as a promise or a contract: "if s, then I guarantee that t".

In which of the following situations can I said to have stood by my contract?

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Situation 1

It was raining and I carried my umbrella.

s is true, t is true

Pondering Implication

s = "It is raining", t = "I carry my umbrella", $s \Rightarrow t$

Think of the implication as a promise or a contract: "if s, then I guarantee that t".

In which of the following situations can I said to have stood by my contract?

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Situation 2

It was raining and I didn't carry my umbrella.

s is true, t is false

Pondering Implication

s = "It is raining", t = "I carry my umbrella", $s \Rightarrow t$

Think of the implication as a promise or a contract: "if s, then I guarantee that t".

In which of the following situations can I said to have stood by my contract?

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Situation 3

It was not raining and I carried my umbrella.

s is false, t is true

Pondering Implication

s = "It is raining", t = "I carry my umbrella", $s \Rightarrow t$

Think of the implication as a promise or a contract: "if s, then I guarantee that t".

In which of the following situations can I said to have stood by my contract?

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Situation 4

It was not raining and I didn't carry my umbrella.

s is false, t is false

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

Truth Table

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

More Truth Tables

p	q	$p \wedge q$	$p \vee q$	$p \Leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

Using Logical Notation

We can rewrite some of our earlier definitions and theorems.

- If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
  - $A \subseteq B \wedge B \subseteq A \Rightarrow A = B$ [Precedence: $\wedge$ before $\Rightarrow$]
- We can also say: if $A = B$ then $A \subseteq B$ and $B \subseteq A$.
  - $A = B \Rightarrow A \subseteq B \wedge B \subseteq A$
- We can combine the two statements above.
  - $A \subseteq B \wedge B \subseteq A \Leftrightarrow A = B$

Verifying Logical Equivalence

I claim that the following two propositions are equivalent, no matter what p and q represent:

"$p \Rightarrow q$" and "$q \vee \neg p$"

How to verify this?

p	q	$\neg p$	$q \vee \neg p$	$p \Rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Important Reading Assignment

Please read Sections 3.1 and 3.2 of your textbook before next class.

Some of the examples refer to things from Chapter 2 in the book. Feel free to skim over those examples if you don't understand them.