

Please think carefully about how you are going to organise your answers *before* you begin writing. Make sure your answers are complete, clean, concise and rigorous.

1. Let L be the language given by the regular expression $(ab \cup aab \cup aba)^*$.
 - 1.1. Design an NFA for L that has no ε -transitions and has only 4 states. [6 points]
 - 1.2. Convert the above NFA into a DFA for L by mechanically using the *subset construction* we studied in class. [10 points]
 - 1.3. Remove all states that are unreachable from the start state of the resulting DFA, to get a 7-state DFA for L . [3 points]
 - 1.4. If you carefully observe this DFA, you will notice two states that can be replaced by a single state. Do this and draw the resulting DFA. Your final DFA should have exactly 6 states. [7 points]
2. Construct NFAs equivalent to following regular expressions (your NFAs may have ε -transitions):
 - 2.1. $10 \cup (0 \cup 11)0^*1$ [7 points]
 - 2.2. $((0 \cup 1)(0 \cup 1))^* \cup ((0 \cup 1)(0 \cup 1)(0 \cup 1))^*$ [7 points]
3. Give regular expressions for the following languages.
 - 3.1. $\{w \in \{0, 1\}^* : w \text{ has three consecutive 0's or three consecutive 1's or both}\}$. [7 points]
 - 3.2. $\{w \in \{0, 1\}^* : w \text{ has three consecutive 0's and three consecutive 1's}\}$. [7 points]
 - 3.3. The set of strings in $\{0, 1\}^*$ with an equal number of 0's and 1's such that no prefix has two more 0's than 1's nor two more 1's than 0's. [10 points]
 - 3.4. Let us define a valid floating point number as $u.v$, where u and v are (finite) strings of decimal digits (0..9) satisfying the following constraints: (the symbol "." between u and v is the decimal point.)
 - i. Neither u nor v may be ε .
 - ii. u can be just 0. If u is not 0, u has no leading 0's.
 - iii. v can be just 0. If v is not 0, v has no trailing 0's.(Thus, for example, 0.0, 231.0 and 5.608 are valid, but 0.00, 05.68, .65, 12. and 4.5100 are not valid.)
Give a regular expression for the set of valid floating point numbers described above. You might want to introduce some notation first to keep your expression small and readable. [10 points]

4. Let L be a nonempty language and M an NFA that recognizes L . Prove that M can be converted into an NFA M' which recognizes the same language L and has exactly one accept state. Your proof *must* describe M' both informally, using plain English, *and* formally, using mathematical notation. [10 points]

5. For a language L over alphabet Σ , define $\text{HALF}(L) = \{x \in \Sigma^* : \exists y \in \Sigma^* (|x| = |y| \text{ and } xy \in L)\}$. Prove that if L is regular, then so is $\text{HALF}(L)$. Your proof *must* be formal; proofs not written in a formal mathematical style get very little credit even if they express the right intuition. [16 points]

Hint: Approach 1: Build an NFA. Nondeterministically guess which state the DFA for L will end up in after reading x and nondeterministically guess a y to append to x . Approach 2: Build a DFA. Work forwards and backwards simultaneously and try to meet in the middle.

Challenge Problems

Remember that challenge problems carry no regular credit, but are intended to provide a higher level of challenge for those who want to think further about the theory of computing.

CP1: For the language L from Problem 1, prove that it is impossible to design a DFA with 5 or fewer states.

CP2: For a language L over alphabet Σ , define $\text{LOG}(L) = \{x \in \Sigma^* : \exists y \in \Sigma^* (|y| = 2^{|x|} \text{ and } xy \in L)\}$. Prove that if L is regular, then so is $\text{LOG}(L)$.