Homework 6 Due Feb 25, 2005

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When describing a Turing Machine, be sure to state explicitly what "extra features" (such as multiple tapes and/or non-determinism) you are planning to use, if any. Feel free to use implementation descriptions, as opposed to formal descriptions, unless explicitly asked to do otherwise. If asked to give formal descriptions, *draw* the transition diagram rather than writing it out as a table.

As usual, please think carefully about how you are going to organise your answers *before* you begin writing. Make sure your answers are complete, clean, concise and rigorous.

- 1. Formally describe a single-tape deterministic TM that decides the language $\{w \in \{0,1\}^* : w = w^{\mathcal{R}}\}$. In addition to drawing the TM's transition diagram, to get **any** credit, you must (1) explain the overall strategy **clearly** in English, and (2) describe what each state of your machine stands for. (Without such explanations and comments, grading would be impossible.)
- 2. Formally describe a two-tape deterministic TM that decides the language $\{w \in \{0,1\}^* : w = w^{\mathcal{R}}\}$. Provide a diagram, plus explanations and comments, exactly as above. You may assume that in a particular move one (or both) of the heads is allowed to remain stationary. Thus, the transition function for such a TM would look like

$$\delta: Q \times \Gamma^2 \longrightarrow Q \times \Gamma^2 \times \{L, R, S\}^2$$
.

When drawing a diagram, if $\delta(q, a, b) = (r, c, d, L, S)$, for example, you would draw an arrow from q to r and label it " $(a, b) \rightarrow (c, d), (L, S)$."

Appreciate the ease of programming with two tapes for this language.

[15 points]

- 3. Formally describe a two-tape NDTM for the language $\{ww : w \in \{0,1\}^*\}$. Again, provide a diagram, plus explanations and comments.
 - Appreciate the ease of programming resulting from nondeterminism and the availability of two tapes.

[20 points]

- 4. Prove that decidable languages are closed under (a) union, (b) intersection, (c) complement, (d) concatenation, and (e) Kleene star. [15 points]
- 5. Prove that recognizable languages are closed under (a) union, (b) intersection, and (c) concatenation.

 Smile smugly as you mentally toss off a proof that they are also closed under Kleene star (but do not turn in a proof of this fact).

 [15 points]
- 6. Show that a k-tape TM M can be simulated by a single-tape TM M' in such a way that a computation which takes time t (i.e., t steps of one configuration yielding another) on M takes time $O(t^2)$ on M'. The big-O notation may hide a constant that depends on k. It may help to use a slightly different multitape-to-single-tape encoding from the one described in class, such as one that interleaves the symbols on the k tapes. [15 points]

Challenge Problems

CP6: Make an appropriate formal definition of a *deterministic* pushdown automaton with *two* stacks. Call such an automaton a 2-S-DPDA. Prove, by formal construction, that every decidable language can be accepted by a 2-S-DPDA.

This is not very hard. You are strongly encouraged to think about it.