Mid-Term Exam
Due by 11:15am on Feb 11, 2009

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Submission Instructions

- Submit exactly like a homework. Late penalties apply in the same way as for a homework.
- Please write or type your solutions neatly and **staple together your sheets of paper**. The course staff is not responsible for sheets lost due to lack of stapling.

Honor Code

- **You must work on the exam alone**; you may not collaborate with anyone.
- You may consult your textbook (Sipser), your notes, and anything posted on the CS 39 website. Consulting anything else (e.g., last year's notes, your friend's notes, other websites) is a violation of the Honor Code.
- You may speak to others about the exam only in *complete generality* (e.g., "The exam is hard", "I'm almost finished with the exam", "I'll be working on the exam tonight"). You may not speak about the exam in any detail whatsoever (e.g., "Problem 3 is hard", "Problem 5 is easy", "That pumping lemma problem is tough").
- Since this is an exam, we (the TAs and the instructor) cannot help you with the particular problems on it, nor can we check if you are on the right track with a problem. However, as you attempt to solve these problems, if you discover that your understanding is not complete on some topics, please see any of us. We are willing to help you with your understanding of the course material to any degree.

Important General Instructions

- You may use, without proof, any results proven in class, in a homework, or in the textbook; simply cite the result you are using. Thus, for example, if you need the result "$\{0^n 1^n : n \geq 0\}$ is a non-regular language" as part of some formal proof you are writing, you may simply cite the appropriate theorem in the textbook.
- Please read each question carefully. If you misread and answer a different question than the one asked, unfortunately, you will not get credit.
- **Good luck!**

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1. Write a regular expression for the language generated by the following CFG:

$$\begin{aligned} S &\longrightarrow AT \\ T &\longrightarrow AAT \mid BBT \mid AA \\ A &\longrightarrow 0 \\ B &\longrightarrow 1 \end{aligned}$$

Keep your expression simple. No justification required. [5 points]

2. Draw a DFA (no justification required, but keep your DFA simple) for the language

$$\{x \in \{0,1\}^* : x \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}.$$

For example, 101 and 0000 are in the language, but 1010 is not. [5 points]

3. Design a CFG that generates $\{0^i1^j : i < j < 2i\}$. Explain the logic behind your CFG by describing what strings are generated by each of your variables. No further proof is necessary. [10 points]

4. Draw a PDA that recognizes $\{a,b\}^* - \{a^nb^n : n \geq 0\}$. Give a complete diagram showing states and transitions, not a formal description. Also provide a brief justification (no need for a formal proof) that your PDA works correctly. [10 points]

All of the remaining problems ask for proofs. While writing a proof, *if* you need to construct an automaton (DFA/NFA/PDA), you *must* specify it formally and you *must* give some justification that your construction is correct, though this justification need not be strictly formal.

5. For a language L, define $\textsc{Rev}(L) = \{x^{\mathcal{R}} : x \in L\}$; recall that $x^{\mathcal{R}}$ denotes the reverse of the string x. Are regular languages closed under the operation $\textsc{Rev}$? Prove your answer. [10 points]

6. For a language L, define $\textsc{Swap}(L) = \{yx : |x| = |y| \text{ and } xy \in L\}$. Prove that if L is regular then $\textsc{Swap}(L)$ can be recognized by a PDA. [15 points]

7. Are regular languages closed under the operation $\textsc{Swap}$? Prove your answer. [10 points]

8. For sets A and B, write "$A \subset_\infty B$" if $A \subseteq B$ and $B - A$ is an infinite set. For example, if $\mathbb{Z}$ and $\mathbb{N}$ denote the set of all integers and the set of positive integers respectively, then we have $\mathbb{N} \subset_\infty \mathbb{Z}$.

Suppose A and B are regular languages such that $A \subset_\infty B$. Prove that there exists a regular language L such that $A \subset_\infty L \subset_\infty B$. [15 points]

A hint for the last problem: It may help to review all the closure properties your know, and then start by proving that every infinite regular language can be partitioned into two disjoint infinite regular subsets.