

Welcome to CS 39

Theory of Computation

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<http://www.cs.dartmouth.edu/~cs39>

Administrivia: Lectures

- We will use *every* X-hour.
- It is *vital* to attend every lecture, or you'll be overwhelmed trying to catch up.
- Almost every lecture will define new concepts
 - Pay attention!
 - Stop me the *moment* you don't follow something
- Lectures will not slavishly follow textbook
 - Take good notes in class
 - Will be helpful during exams, and later in life

Administrivia: Grading

- HW: 35, Quizzes: 15, Midterm: 20, Final: 30
- Homework given out Wed, due next Wed
 - HW due at *start* of class (late policy: see web)
 - Will require plenty of “think time”
 - Start early!
- Two in-class quizzes (Week 4 & Week 8)
- Take-home midterm, take-home final

Administrivia: Honor Code

- Homework
 - You can work on homework problems in groups.
 - But when you start writing your solutions you must work alone and write in *your own words*.
 - If you borrowed a key idea for a solution from your classmate(s), acknowledge them like a true scholar.
- Exams
 - You may not discuss exam problems with *anybody*, not even with the TAs or me.

Administrivia

- If you need card access to Sudikoff on evenings (after 6pm) and weekends, see Christine Claudino in the front office.
- If you have a documented disability and need academic adjustments, please see me privately before Jan 16.

A basic theme of CS 39

- “Computers cannot solve everything.”
- Perhaps this seems obvious to you, as stated:
 - How do I become a millionaire?
 - How do I unite quantum theory and gravitation?
 - How do I find out if God exists?
- These “problems” are not well-posed.
 - Or, these are not “computational problems.”
- A well-posed problem should describe the desired output for every possible input.

What CS 39 is about

- What can or cannot be computed?
 - What does “to compute” mean?
 - What is a “computer”?
 - Are there really things that cannot be computed?
- How quickly can something be computed?
 - Will you give me the latest snazziest computer?
- Abstract *models of computation*

What CS 39 is about

- Finite automata
 - Regular languages
- Push-down automata
 - Context-free languages
- Turing machines
 - Decidable languages (and undecidable ones)
- Polynomial time bounded Turing machines
 - P, NP, NP-complete

Okay, what CS 39 is *really* about

Proofs,
Proofs, proofs, proofs,
Proofs, proofs, proofs

And...

More proofs!

Relax

Although this is a mathematical course...

- No derivatives or integrals
- No binomial coefficients
- No number-crunching calculations
- No probability theory

Just *mathematical notation*
and the *power of reasoning*.

Infinite Loop Tester

- You are a grader for CS 5
- Students submit a program “foo.java”
 - You have test input files “1.inp”... “100.inp”
 - Every test case correctly handled: 1 point
 - Infinite loop: -20 points (penalty)
- You wish to automate the grading process
 - Write a program “ILT.java” which takes two input files ---“foo.java” and “i.inp” --- and checks whether “foo.java” enters an infinite loop on input “i.inp”

It's impossible

- The Infinite Loop Tester cannot be written!
- This is not because of any deficiency or peculiarity of the Java programming language, the task simply cannot be done by a computer.
- Truly remarkable that a well-posed *computational* problem cannot be solved by a computer!

Hilbert's Tenth Problem

- Given a system of polynomial equations with integer coefficients, like

$$\begin{aligned}x^2 - 3xz &= 5 \\ 7xy + 4y^3 - xz^2 &= 2\end{aligned}$$

- is there a solution in the integers?
- i.e., in this case, are there integers x, y, z that satisfy both these equations?

It's impossible

- A computer program that solves Hilbert's Tenth Problem cannot be written.
- Again, it does not matter what type of computer or what programming language you use.
- Proof beyond the scope of this course.
(But contact me later if you're curious.)

A Goal

One of the main goals of CS 39 is to develop a theory that enables us to prove such results.

What's Next

- Overview of mathematical notation
 - Set theory
 - Logic
- Overview of proof writing
- On to the main course

Math Notation: Sets

- $A = \{1,2,3,4,5\}$ $B = \{1,3,5,7,9\}$
- $|A| = 5$
- $A \cup B = \{1,2,3,4,5,7,9\}$
- $A \cap B = \{1,3,5\}$
- $A \times B = \{(1,1), (1,3), (1,5), \dots, (5,7), (5,9)\}$
- $A - B = \{2,4\}$
- $2^A = \{\emptyset, \{1\}, \{2\}, \dots, \{1,2\}, \{1,3\}, \dots, \{1,2,3,4,5\}\}$

More Math Notation

- $A = \{1,2,3,4,5\}$ $B = \{1,3,5,7,9\}$
- $\{x \in A \cup B : x \bmod 3 = 1\} = \{1,4,7\}$
- For integer n , let $S(n) = \{n, n^2\}$
- $S(9) \cup S(10) = \{9,81,10,100\} = \{9,10,81,100\}$
- $\bigcup_{i=1}^3 S(i) = \{1,1\} \cup \{2,4\} \cup \{3,9\} = \{1,2,3,4,9\}$
- $\bigcup_{n \in B} S(n) = \{1,3,5,7,9,25,49,81\}$
- $\{S(n) : n \in A\} = \{\{1\}, \{2,4\}, \{3,9\}, \{4,16\}, \{5,25\}\}$