Approximation Algorithms

EO249

(Problem Set 4) Due: Mar 6th, 2015

Solutions need to be submitted by email to eo249iisc@gmail.com. We prefer latexed solution. Before giving the solutions, you should write how many problems you have attempted and how many you think you have solved. Starred problems are optional and (possibly) more fun.

Exercise 1. Design a 4-approximation for the metric facility location problem by modifying the algorithm done in class.

Exercise 2. (Integrality gap of Set cover LP) Fix a positive integer f. Design an instance of the set cover problem where the frequency of each element is $\leq f$, but the optimal set cover is at least $(f - \varepsilon)$ times the value of the LP-solution for any $\varepsilon > 0$.

Exercise 3. (Integrality gap of GAP) Recall the LP relaxation of GAP done in class. We proved that the integrality gap is $\geq 1/2$. That is, given any instance, the profit obtained by our algorithm was at least $\frac{1}{2} \cdot LP_{GAP}$. Prove that the integrality gap is $\leq 1/2 + \varepsilon$ for any $\varepsilon > 0$. Caution: you are not allowed to construct an instance with $w_{ij} > B_i$ for any i, j.
Hint: First try to prove integrality gap $\leq 2/3$. Then try $\leq 3/5$. Then ...

Exercise 4. (Integrality gap for matchings) In class, we proved that the maximum profit **bipartite** matching LP has integrality gap 1, that is, for any LP solution we can find a matching of profit more than the LP value. Write an IP and LP relaxation for the maximum profit matching in a **general** graph. Discuss the intergrality gap of the relaxation.

Exercise 5. (*) Design an $O(\log n)$-approximation algorithm for the general facility location problem (where the costs may not form a metric).