Problem 1. Given any tree, prove that there are an even number of vertices whose degree is an odd number. Do the number of vertices with even degree need to be even as well?

Problem 2. We are given a directed graph \( G = (V, E) \) with non-negative costs \( c(e) \) on every edge. There is a source vertex \( s \) and a destination vertex \( t \). Give an efficient algorithm for finding the path from \( s \) to \( t \) with the smallest total cost, which visits at most 10 other vertices on the way.

Problem 3. Which of the following are NP hard, and which have polynomial time algorithms? Answer with either a reduction or an algorithm.

- Given a graph \( G \), decide whether or not it contains a cycle of odd length.
- Given a set of \( n \) integers in the range \([-n, +n]\) and a target \( B \), decide whether or not there is a subset of these which add up to exactly \( B \).
- Given a set of \( m \) linear equations in \( n \) variables and integer coefficients in the range \([-n, +n]\), decide whether or not
  a) there is an assignment of the variables to reals which satisfy all the equations.
  b) (*) there is an assignment of the variables to \( \{0, 1\} \) which satisfy all the equations.

Problem 4. (*) Given a directed graph \( G = (V, E) \) with costs \( c(e) \) on edges, a cycle cover is a subset \( F \subseteq E \) of the edges such that (a) each component of \( (V, F) \) is a cycle, and (b) each vertex of \( V \) is in exactly one cycle. The cost of the cycle cover is the sum of the costs of the edges in \( F \). Design a polynomial time algorithm to find the minimum cost cycle cover of a graph \( G \).

Hint: Consider the bipartite graph \( H \) with bipartition \( A \cup B \) where \( A \) and \( B \) are copies of vertices of \( G \), and there is an edge from \( u \in A \) to \( v \in B \) if and only if (iff) there is a directed edge from \( u \) to \( v \) in \( G \). What do cycle covers in \( G \) correspond to in \( H \)?