Kernel Nyström Method for Light Transport

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Reconstruct light transport matrix from small number of acquired images

Exploits linear and nonlinear data coherence

Adaptive measurement techniques
Kernel Nyström Relighting

Effectively reconstructs light transport matrix to render new lighting conditions.
Outline

1. Introduction
   - Mathematical Definition
   - Related Work

2. The Kernel Nyström Method
   - Asymmetric Generalization
   - The Kernel Extension
   - Adaptive Light Transport Measurement

3. Conclusion
   - Results
   - Future Work
Mathematical Definition

\[
\begin{bmatrix}
V \\
\end{bmatrix} =
\begin{bmatrix}
T \\
\end{bmatrix} \cdot
\begin{bmatrix}
L \\
\end{bmatrix}
\]

\[ V : \text{outgoing radiance seen by camera, } m \text{ pixels} \]
\[ T : m \times n \text{ light transport matrix} \]
\[ L : \text{vector of incident radiance from } n \text{ light sources} \]

(Ng et al. 03, Peers et al. 09)
**Mathematical Definition**

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\begin{bmatrix}
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- \( V \): outgoing radiance seen by camera, \( m \) pixels
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- \( L \): vector of incident radiance from \( n \) light sources

(Ng et al. 03, Peers et al. 09)
PREVIOUS METHODS

Three main categories:

- Brute Force
- Sparsity Based
- Coherence Based
**Previous Method: Brute-Force**

Debevec et al., Light Stage

- Massive array of lights
- Capturing thousands of images

(Relighting Human Locomotion.Debevec et al., 06)
Previous Method: Sparsity-Based

Peers et al.

- Project Haar wavelet noise to infer reflectance matrix
- Does not handle complex occlusions

(Inferring reflectance functions from wavelet noise. Peers et al., 05)
Previous Method: Coherence-Based

Fuchs et al.
- Adaptive sampling scheme for reflectance field
- Exploits coherence in either rows or columns
Reconstructs low-rank symmetric matrix from sparsely sampled columns; accurate when rank($T$) $\leq k$

$$T = \begin{bmatrix} A & C^T \\ C & B \end{bmatrix} \approx \begin{bmatrix} A & C^T \\ C & CA^{-1}C^T \end{bmatrix}$$

$(k+n) \times (k+n)$

(Williams and Seegor 00)
Asymmetric light transport matrices with image pixels as columns and light sources as rows
**The Asymmetric Nyström Method**

If \( \text{rank}(T) = \text{rank}(A) \), we can expect the following relationships:

\[
\begin{bmatrix}
    C & B
\end{bmatrix} =
\begin{bmatrix}
    P & A \quad R
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
    R & B
\end{bmatrix} =
\begin{bmatrix}
    A & C
\end{bmatrix} Q
\]

implying \( C = PA, \ R = AQ, \) and \( B = PR = CQ. \) Thus,

\[
B = PAQ = PAA^+AQ = CA^+R
\]

\( A^+ \) denotes the Moore-Penrose pseudoinverse of \( A \) with property \( AA^+A = A \) and is found by applying SVD to \( A \).
**The Asymmetric Nyström Method**

\[
T = \begin{bmatrix} A & R \\ C & B \end{bmatrix} \approx \begin{bmatrix} A & R \\ C & CA + R \end{bmatrix}
\]

Works under assumption: \( \text{rank}(T) = \text{rank}(A) \)
Standard approach for enhancing performance of machine learning algorithms based on nonlinear transformations of input.

Map vectors in the data space to a higher dimension feature space.

(Cristianini and Shawe-Taylor 00)
The Kernel Method

\[ f(T) \approx K = \begin{bmatrix} f(A) & f(R) \\ f(C) & f(C)(f(A)) + f(R) \end{bmatrix} \]

\[ T \approx f^{-1}(K) \]

See paper for more details on how \( f \) is a kernel method.
The Light Transport Kernel

Use the nonlinear power function

\[ f(x) = x^\gamma \]

- One parameter, optimal function easy to find
- Produces enhanced reconstructions
We use golden section search to find optimal $\gamma \in_{\log} [0.001, 1000]$ that minimizes

$$g(\gamma) = \frac{\|f(A)\|_2^*}{\|f(A)\|_2} \int_0^1 \frac{1}{f'(x)} p(x) \, dx ,$$

$$\|X\|_* = \sum_i \delta_i , \quad \|X\|_2 = \max_i \{\delta_i\}$$

where the $\delta_i$'s are singular values of X and $p$ is the distribution density of values in A (assumed to be identical to that in T).

Optimization takes only few seconds.

(Press et al. 92).
See paper for more details.
Acquisition Equipment Setup

Figure 4: Device setup for capturing sparse columns and rows of the light transport matrix. (a) Illustration. (b) Photograph.

A device setup is designed for acquiring a batch of rows and columns, which is represented by a very large transport matrix on a 2D plane to image pixels. Before capturing, we calibrate the color and intensity of each point light source. In our implementation, we first calibrate the point light source positions and pixels of the dual camera, and then down-sample the sampled column and row images.

After calibration, we place the scene objects on the other point light sources within the regular grid by interpolating under the diffuser plane and start to capture the columns of the light transport matrix. A stratified sampling scheme is applied for sampling rows of the matrix based on sparsely sampled point light sources sampled on 40 regular grids by capturing the images with the dual camera simultaneously to build the correspondence between the dual camera and kernel estimation. We then employ a typical acquisition session (including image acquisition, HDR reconstruction, and kernel estimation) takes about 135 minutes for capturing 150 rows and 150 columns from the scene. The image transport from point light sources on a 2D plane to image pixels is measured via a dual setup by exploiting the reciprocity of light. For each row (i.e., pixel location) to be sampled, we adjust the distance of the laser emitters. We utilize the sampled column values to guide the row sampling.

Similar to [Hašan et al. 2007], we pack the cluster center vector as a sampling pixel. For each cluster, the vector closest to the mean of the cluster is selected as the sampling pixel. We then employ the sampled column and row images. (d) Photograph of the scene in row sampling. (b)(c) Two extended sample sets.
Light Transport Measurement

Device setup

Light Transport Matrix, A

Unsampled Matrix
Light Transport Measurement

Device setup

Light Transport Matrix, A

Unsampled Matrix

Sampling Image Pixels
Light Transport Measurement
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Future Work
Thank you!

Any questions?