\[
[S; \varepsilon \rightarrow] \rightarrow [A; \varepsilon \rightarrow] \quad [C; \varepsilon \rightarrow] \rightarrow [B; e \rightarrow]
\]
\[
[S; \varepsilon \rightarrow] \rightarrow A[D; \varepsilon \rightarrow] \quad [C; \varepsilon \rightarrow] \rightarrow B[E; \varepsilon \rightarrow]
\]
\[
[S; \varepsilon \rightarrow] \rightarrow AD \varepsilon \rightarrow [1] \quad [C; \varepsilon \rightarrow] \rightarrow BE \varepsilon \rightarrow [4]
\]
\[
[A; \varepsilon \rightarrow] \rightarrow a[C; \varepsilon \rightarrow] \quad [B; e \rightarrow] \rightarrow bced \varepsilon \rightarrow [3]
\]
\[
[A; \varepsilon \rightarrow] \rightarrow aC \varepsilon \rightarrow [2] \quad [E; \varepsilon \rightarrow] \rightarrow e \varepsilon \rightarrow [6]
\]
\[
[D; \varepsilon \rightarrow] \rightarrow \varepsilon \rightarrow [5]
\]

(16)

It is, of course, unnecessary to list productions which cannot be reached from \( [S; \varepsilon \rightarrow] \). Condition (15) is immediate; one may see an intimate connection between (16) and the tree (3).

Our second method for testing the \( \text{LR}(k) \) condition is related to the first but it is perhaps more natural and at the same time it gives a method for parsing the grammar \( \mathcal{G} \) if it is indeed \( \text{LR}(k) \). The parsing method is complicated by the appearance of \( \varepsilon \) in the grammar, when it becomes necessary to be very careful deciding when to insert an intermediate symbol \( A \) corresponding to the production \( A \rightarrow \varepsilon \). To treat this condition properly we will define \( H_k'(\sigma) \) to be the same as \( H_k(\sigma) \) except omitting all derivations that contain a step of the form

\[ A \omega \rightarrow \omega, \]

i.e., when an intermediate as the initial character is replaced by \( \varepsilon \). This means we are avoiding derivation trees whose handle is an empty string at the extreme left. For example, in the grammar

\[
S \rightarrow BC \varepsilon \rightarrow \varepsilon, \quad B \rightarrow Ce, \quad B \rightarrow \varepsilon, \quad C \rightarrow D, \quad C \rightarrow De, \quad D \rightarrow \varepsilon, \quad D \rightarrow d
\]

we would have

\[
H_3(S) = \{ \varepsilon \varepsilon \rightarrow \varepsilon, \varepsilon c \varepsilon \rightarrow \varepsilon, \varepsilon c e \varepsilon \rightarrow \varepsilon, \varepsilon c e d \varepsilon \rightarrow \varepsilon, d e \varepsilon \rightarrow \varepsilon, d e c \varepsilon \rightarrow \varepsilon, d e c d \varepsilon \rightarrow \varepsilon, d e c e \varepsilon \rightarrow \varepsilon, d e c e d \varepsilon \rightarrow \varepsilon, d e c e d c \rightarrow \varepsilon \}
\]

\[
H_3'(S) = \{ d e c \varepsilon \varepsilon \rightarrow \varepsilon, \varepsilon d e c \varepsilon \rightarrow \varepsilon, \varepsilon d e c d \varepsilon \rightarrow \varepsilon, \varepsilon d e c e \varepsilon \rightarrow \varepsilon, \varepsilon d e c e d \varepsilon \rightarrow \varepsilon, \varepsilon d e c e d c \rightarrow \varepsilon \}.
\]

As before we assume the productions of \( \mathcal{G} \) are written in the form (11). We will also change \( \mathcal{G} \) by introducing a new intermediate \( S_0 \) and adding a "zeroth" production

\[
S_0 \rightarrow S \varepsilon \rightarrow^k
\]