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Count Him In

Math? It's All Around to Be Found. One Writer Is Positive He's Equal to the Task.

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Math is hot. The TV show "Numb3rs," featuring a crime-solving mathematician, is a hit. In the past few years there has been a run of popular math movies, including "Pi," "Good Will Hunting" and "A Beautiful Mind," the Russell Crowe film about Nobel Prize-winning mathematician John Nash that grossed more than $170 million.

The truth is, math has been hot for eons. It has given civilization, among other things, time, distance, weight, currency, commerce, computers, "Sesame Street," speedometers, the NFL, Pixar, Yahoo!, iPods and "The Da Vinci Code." It makes life easier, more manageable and more orderly.

Except when it comes to the problems that can't be solved.

Professor Dan Rockmore, on the terrace of his Upper East Side apartment, sees math problems wherever he looks, some much easier to solve than others. (Photos By Helayne Seidman For The Washington Post)

Dan Rockmore is fascinated by just such a problem. He's 43, an easygoing, wire-haired professor of mathematics at Dartmouth College and author of the just-published "Stalking the Riemann Hypothesis: The Quest to Find the Hidden Law of Prime Numbers."

The Riemann hypothesis is one of the Seven Millennium Problems posed by the Clay Mathematics Institute in Cambridge, Mass., and whoever proves it will win $1 million. The seven, writes Stanford mathematician Keith Devlin, are "the hardest and most important unsolved mathematics problems in the world; they have resisted numerous attempts at solution, over many years, by the best mathematical minds around."

So far, none has been conclusively solved. But that doesn't keep mathematicians from trying.

To understand how someone can spend hours, days, years wrestling with an insoluble problem, you have to look at the world through a mathematician's eyes. That's where Rockmore comes in. He's not one of those fluky-flakey number nerds you read about.

He's a hiker, a tennis player, a distance runner. He's got a loving family, a Manhattan *pied-à-terre* and patience enough to explain math to the unmathematical. He is an expositor who scored higher on his verbal SATs than on his math and he has agreed to spend the afternoon walking you through some of the toughest concepts in math -- literally.

Over lunch at Il Mediterraneo, near his apartment on t*he* Upper East Side, he begins with a piece of cake. Say that *you* and he want to split a single piece of cake. You each want a fair share, so you agree that Dan will slice the cake and you will choose the half you want. Dan cuts it down the middle and you take the piece that you *think* is slightly larger. Now Dan feels like he has gotten an equal portion -- he cut it in half, remember -- and, because you were given a choice, you believe that you have gotten slightly more than Dan got. Miraculously, the two halves will seem to add up to more than a whole.

This is known as the Mathematics of Envy and it's only one small way that a mathematician *tries* -- tries -- to make sense of this complex and perplexing world.

Mathematicians move around the world in different ways from the rest of us. They live in a parallel reality -- seeing numbers where we see words, equations where we see poetry. "In order to understand the universe," Galileo wrote in the 17th century, "you must know the language in which it is written. And that language is mathematics."

Rockmore gestures toward wine bottles that are stored, bottoms out, in cubbyholes above the cafe bar. "I see circles and polyhedra," he says.

On this clear blue, purified spring day, Dan takes a postprandial stroll and Manhattan becomes a three-dimensional chalkboard. Between the geometry of architecture and calculus of urban life, you begin to see the sidewalks and the skyscrapers through a mathematician's eyes and somewhere along the way, theoretical math becomes, well, more concrete.

A sunflower at a florist's shop helps illustrate Fibonacci numbers. A stack of tomatoes at a greengrocer suggests Kepler's Conjecture. A stand of seven trees leads back around to a conversation about the Riemann Hypothesis. It's like taking a tour of a familiar place with a foreign-tongued guide.

Things You Can Count On

Dan Rockmore has long been enamored of numbers. He remembers as a little boy, walking with his physicist father to buy a newspaper in the morning. He talked to his dad about fractions; sometimes he counted his steps. Decades later he speaks and writes of math -- and the beauty of proofs -- with the reverence of a loving son. "We count. That's what we do," he says. "I have always assumed that everybody did it."

We are also, he says, hemmed in by math. "Life is about being creative within bounds," Rockmore says. "You can be infinitely creative, but there are some hard and fast rules."

Often those rules are represented by numbers. You only have two hands. There are only 24 hours in a day. The alphabet has 26 letters; the major musical scale seven notes.

In "Stalking," he writes that the natural numbers -- the plain old numbers we use every day, such as one, two, three, four, etc. -- seem to have been with us from the beginning of time. They "are implicit in the journey of life, which is a nesting of cycles imposed upon cycles, wheels within wheels. One is the instant. Two is the breathing in and out of our lungs, or the beat of our hearts. The moon waxes and wanes; the tides ebb and flow. Day follows night, which in turn is followed once again by day. The cycle of sunrise, noon and sunset gives us three. Four describes the circle of the seasons."

Though you won't have to count much in this article, you must understand that there are two types of natural numbers: composites and primes. Composites, such as 4, 6, 8, 9, 10, 12 and so on, can be divided by smaller numbers. The primes -- 2, 3, 5, 7, 11, 13, 17 and so on -- cannot be divided by smaller numbers, except 1. Prime numbers have a practical application these days; we use them in e-commerce to encrypt digital information, which makes it harder for identity thieves to steal our Social Security and credit card numbers on the Internet.

Mathematicians were intrigued by the primes long before there were computers. Euclid, a 3rd-century B.C. mathematician in Greece, pointed out that there are an infinite number of primes. Leonhard Euler, an 18th-century Swiss mathematician, discovered that primes appear in certain series. C. F. Gauss, an early 19th-century German known as "the prince of mathematics," tried to figure out why the primes are farther and farther apart as you count higher and higher. And in 1859, German genius Georg Friedrich Bernhard Riemann put forward a hypothesis that prime numbers occur -- along that never-ending number line -- in a certain pattern. He concocted a formula that helps to predict when the next prime will occur. Today it is called the Riemann Hypothesis.

It has not been proved, beyond the shadow of a doubt, that Riemann's formula can predict the primes all the way out to infinity. That is why mathematicians still fiddle with the hypothesis and why Rockmore wrote his book.

Pointing to a sparkling yellow-and-black sunflower at Apple Tree Flowers on the corner of 69th Street and Second Avenue, Rockmore speaks of Leonardo Pisano Fibonacci, a 13th-century Italian mathematician. Fibonacci is famous for figuring out a special sequence of natural numbers. He began with 0, then 1. From then on, he added the previous two numbers to find the next. For example, 1, 2, 3, 5, 8, 13, 21, 34. A simple-enough pattern -- called the Fibonacci series -- but it becomes profound when you discover that the numbers pop up throughout the natural world: in certain flower petals, pine cones and the seed head of a sunflower, where the number of spirals -- usually 34 or 55 or 89 -- allows nature to pack as many seeds as possible into a circle.

A few blocks away, Rockmore pauses in front of a display of vegetables at the Garden Deli. The tomatoes are stacked neatly, one layer latticed atop another. Mathematicians marvel at the ways spheres fill up space, he says. Johannes Kepler asserted in 1611 that this way of stacking -- called "face-centered cubic packing" -- is the most efficient, but for centuries no one could prove it. A proof is a very detailed, logical process for verifying a mathematical assertion.

In 1998 Thomas Hales at the University of Pittsburgh posted a proof of the Kepler theorem online that, assuming that one is dealing with perfect spheres and perfect cubes, was eventually accepted by the math community.

The Green Space Theorem

To really contemplate natural numbers and everyday math, Central Park is the place to go. Historically, great math problems and solutions have sprung from walks in a park. Euler, for example, pondered a famous conundrum while strolling through the parks of Konigsberg. Two rivers flowed through the East Prussian town, among two islands and the mainland. All in all, there were seven bridges connecting three pieces of land. Townsfolk made a game out of trying to cross all seven bridges during a single walk -- without backtracking.

As mathematicians often do, Euler took the fun out of the game. First he turned the problem into a diagram of networks (lines representing the paths) and nodes (dots representing the land masses). An odd node had an odd number of lines jutting out from it; an even node had an even number of lines. Euler then showed that it was impossible to walk in a continuous circuit -- without retracing your steps -- unless the diagram had no odd nodes or two odd nodes. Since the Konigsberg Bridge Problem had four odd nodes, it proved impossible.

While sitting on a bench, Rockmore tells the story of the 20th-century Hungarian mathematician George Polya, who was in Zurich for some years. Meandering through a park one afternoon, Polya kept running into a colleague and his girlfriend. The colleague believed that Polya was making contact on purpose. Perhaps eager to prove that he wasn't hitting on his friend's girl, Polya devised the Random Walk Problem. He eventually published a mathematical proof showing that if you walk around enough in an infinite grid, you will return to the same points over and over.

Gesturing toward a clump of seven trees in Central Park, Rockmore also returns to certain points. Seven, of course, is a prime. And primes remind Rockmore of Riemann and the seven Millennium Problems.

A Russian math whiz named Grigori Perelman posted a solution in 2002 to one of the problems -- the Poincare Conjecture. The conjecture, crafted in the early 1900s, asks if the properties of the two-dimensional surface of a sphere behave in the same way as the properties of a three-dimensional surface of a sphere, which is something we cannot see but can only imagine.

James Carlson, president of the institute, says that after two years of scrutiny by the professional community, Perelman's proof "still looks good." The institute's board of advisers will have to agree before he gets the money.

The seven problems are incredibly dense. Rockmore says he doesn't completely understand the Yang-Mills Theory, which is the mathematical theory underlying quantum physics. Another of the problems -- the Navier-Stokes Equations that would explain the ways that fluids flow -- is "not well defined," Rockmore says.

Readable explanations of the problems can be found in Devlin's book, "The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time. " The problems are also listed on the Clay Mathematics Institute Web site, *http://www.claymath.org* .

Some of the problems, Rockmore says, may be impossible to solve. "Maybe the Riemann Hypothesis turns out to be wrong," Rockmore says. "Some poor schmo may prove that. But he wouldn't get the million dollars."

Addition and Subtraction

There is much about mathematics that is unclear. For centuries, theoretical mathematics coexisted with uncertainty. "The literature is filled with incorrect theorems in the early 19th century," Rockmore says, but later in the century mathematicians began to insist on more precision.

Since then there has been more pressure to be precise, even as the world has become harder and harder to explain. But the occasional imprecision of mathematics remains a constant. "The zeitgeist affects mathematics," Rockmore says.

And mathematics affects the zeitgeist. Computers have given us the puffed-up notion that we can quantify just about anything -- even love. Rockmore points to eHarmony.com, the online matching service that relies on 29-question surveys to pair compatible people. As the Web site explains it: "The eHarmony.com compatibility matching models were created using factor analyses, multiple regression and discriminant analyses on data gathered from married couples." In other words, feelings are distilled to factors; attraction to analysis.

Back at the restaurant, you and Dan were given three pieces of bread. They brought to mind the Mathematics of Guilt, as described by British mathematician Rob Eastaway in "Why Do Buses Come in Threes? The Hidden Mathematics of Everyday Life." In Eastaway's example, the vicar's wife invites five friends over for tea. She offers her guests a plate of biscuits -- four are chocolate and one plain. All the guests like chocolate biscuits. The first guest takes a chocolate biscuit. So do the second and third. The fourth person knows that if she chooses the last chocolate biscuit, the fifth person will be forced to eat a plain one. She feels guilty and so doesn't take either one. The question, Eastaway asks, is: Should the other guests also have felt guilty, and if so, how do you decide mathematically the amount of guilt each guest should feel?

But guilt is not always based on taking the last biscuit. And love is not always founded on compatibility. Even the piece of cake that two people share might have a blemish on one half or a frayed corner on the other.

In the end, Rockmore decides not to have dessert after all. He orders a cup of coffee instead. Sometimes he stays up all night, trying to solve a problem.