Objects, Arrows and Rectangles

No matter what questions I asked my first thesis advisor, the response always seemed to be the same. At the end of an hour or so the blackboard would contain the same few symbols: an "A", a "B", maybe even a "C" if he were feeling particularly effusive. Always there were arrows, some flowing from letter to letter, others curving back on themselves. They indicated connections, functions, influences. In spite of my detailed and specific questions, ("Should this really be a minus sign? How does this equation follow from the one above?") in the end, I knew that his answer would be the same; shaking his head, telling me that my worries were just "details" and then slowly, patiently leading me back to an explanation of symbols and arrows. I kept asking him about all the different trees, he kept trying to tell me about the forest.

Seeing the forest for the trees, that's how science makes progress and mathematics is no exception. Many, many examples get worked out (the details) and from this a theory slowly emerges, unifying the known examples as specific instances of a general framework. The theory distills the examples to their essence, or as we often say, abstracts the examples, by freeing them from the distractions of their particular setting. The more seemingly disparate the examples, the more powerful the theory connecting them.

In many ways, abstraction in modern art seems to mirror abstraction in mathematics – the goal of distillation, seeking out and representing the essence of a body of work or subject matter. This "problem" has generated a variety of “solutions”, among which the work of Mark Rothko has been, in the eyes of some, pre-eminent. In a famous letter to the New York Times written by Rothko and Adolph Gottlieb in response to a review of one of the first group shows of Abstract Expressionist work, they write that “...we favor the simple expression of a complex shape...” and that “...the progress of a painter’s work...will be towards clarity: toward the elimination of all obstacles between the painter and the idea, and between the idea and the observer” [1]. Rothko’s rectangular forms, floating bodies of color and emotion, has been viewed by some critics (see eg. [2]) as the ultimate generalization of classical figure or landscape portraiture. Each such painting stands for and comprises all landscapes that precede it. These works, like “Orange and Lilac Over Ivory”, reproduced here, are in this way universal portraits – standing for all portraits at once.

Rothko often spoke of the “drama” of these paintings. This is a landscape, but of what? Of the world around us or within? Just as my questions transformed the meaning of my adviser’s letters and arrows, the problems that we bring to our experience of Rothko’s painting will transform our interaction with it. The painting is at once actor and stage, drawing out the viewer in a dramatic gestalt encouraged by the large scale format of paintings like “Orange and Lilac over Ivory”. This is the tension; this is the drama.

In short, this approach is a powerful abstraction, for it does more than rename portraiture, it re-imagines the subject. Rothko’s abstraction is one that is rife with potential, possibility and mystery.

In mathematics too, the powerful abstractions are the ones that create possibility. Our everyday whole numbers are abstractions of equinumerous piles of rocks or bundles of sticks. Think of how much faster you can compute $101 + 83$ than you would be able to take a pile of 101 rocks and 83 rocks, mix them together and count the new pile. Our theory of addition, and the abstraction of whole numbers, gives a technique which works for any number of piles of rocks. But even more, by removing the concrete representation of numbers by objects, we can ask questions like "What is a number, call it x, that has property that $9+x = 4$?", or "What is a number, call it y, that has property that $9 + y = 9$?", and in so doing, give birth to the abstract and useful notions of negative numbers and zero and the world of arithmetic.

And we can go on. The whole numbers, their negatives and zero comprise the integers and we can study this system and its structure as we combine these elements according to the simple rules of addition. In doing this we may see that other systems have things in common with the integers. For example, consider all the ways that you can shuffle a deck of cards. We notice that if we follow one shuffle of a deck by another shuffle, then we still get a shuffle – so that we can combine two shuffles into one shuffle, just as we combine two integers into one by adding them. In this regard the "null" shuffle, which moves no cards at all, acts like zero, and just as any number has its negative, or additive inverse, any shuffle has an "inverse" which has the effect of undoing the original shuffle. In this way, we see that it is possible that some of the understandings that we have of the additive integers may give us insight into card shuffling, and conversely. It is then natural to study the common essence of these two seemingly completely different ideas in order to understand them better.

This approach relates shuffling to the whole numbers through the lens of arithmetic and algebra, but it might be more appropriate instead to consider shuffling simply in terms of its effect on a deck of cards. Shuffling is a natural "symmetry" of a deck of cards, in the sense that after applying a shuffle we still have a deck of cards (albeit in a potentially new order) and so we can ask are the integers in any way a symmetry of some object? A quick look at the frieze on a wall of a Greek ruin may provide a quick answer of yes as we see that by translating the pattern a fixed amount in one direction or another gives the same pattern back. Both the algebraic or operational point of view above, and this geometric point of view of symmetry lead us to the notion of an abstract "group", a concept which turns out to be central to the mathematics of particle physics and the understanding of the fundamental forces of nature. In both cases the abstraction provides the bridge by which the two seemingly disparate ideas are linked, and in giving birth to this new subject tools arise which then further make possible new and never before imagined discoveries. [5]

At the pinnacle of abstraction sits the subject of "category theory", developed by Sammy Eilenberg and Saunders Maclane in the 1940's. The very language of the subject seems purged of specific reference. In category theory we study "objects" and the "arrows" that transform one object into another.

Objects are in fact entire subject areas: groups, rings (areas of algebra), topological spaces (a generalization of shapes), geometries (an abstraction of the very space we live in). Arrows correspond to constructions that may transform one kind of object into another. As above, the theory grew out of the goal of collecting together various common and useful phenomena into a common framework. This time it was the realization that a certain procedure for transforming a topological problem (i.e., one about shapes and their "smooth" deformations) could be translated or transformed into an algebraic one, and that the process for doing this in fact made sense in a variety of situations (see eg. [6,7]). In a sense, all mathematicians are applied category theorists.

Eilenberg has been called "our greatest mathematical stylist". Both he and Rothko saw abstraction as a means for re-imagining a subject and not an end in itself. As the mathematician Peter Freyd remarked in a collection of reminiscences on Eilenberg's life, "He and Maclane had invented the subject, but to them it was always an applied subject, not an end in itself...its inevitability would be based...on the theorems whose proofs required category theory."[4] Alex Heller further goes on to say that "Sammy considered that the highest value in mathematics was to be found...in providing the definitive clarity that would illuminate its underlying order. ..Category theory was him a tool – in fact a powerful one – for expanding our understanding." [4]

Rothko too was driven by the goal of bringing together seemingly unrelated ideas in order that something new might grow. Of course Rothko was not looking to give voice to a new mathematical idea, but instead worked so that some previously ineffable emotion find expression. In his own words, "I do not believe that there was ever a question of being abstract or representational. It's a matter of ending the silence and solitude, of breathing and stretching one's arms again." [4] Rothko's canvases are protean masses of color that initiate a journey, rather than marking one's end. It's a geometric reduction, but a poetic one of mystery and possibility. This is not the literal utopian geometry of Mondrian or Malevich's infinitely precise lines and perfect shapes, but rather its knowable poetic and tragic real world shadow, flickering as if cast on the walls of Plato's cave. This is a geometry of ever-shifting forms with soft, blurred and fuzzy boundaries, a geometry of human experience.

There are some striking similarities between Rothko and Eilenberg. Both arrived in the United States as Jewish refugees, driven here by persecution in their homelands. They had in common a passion for primitive art – Rothko using it as material for his early work, while Eilenberg became one of the world's most famous collectors of Southeast Asian sculpture, ultimately donating his entire collection to the Metropolitan Museum. Almost simultaneously over the World War II era they each gave birth to their most important ideas, and in doing so, created something universal at a time when the world was being torn apart by those who would focus on differences. They made their own worlds whole, by seeing beyond the details that distinguish, to reveal and elucidate elemental and unifying truths in an irreducible shared essence of primordial objects, arrows, and floating rectangles.

References

[1] A. Gottlieb and M. Rothko, Letter to the New York Times.

[2] A. C. Chave, “Mark Rothko: subjects in abstraction”, Yale University Press, 1989.

[3] M. Rothko, “The Romantics Were Prompted”

[4] H. Bass, H. Cartan, P. Freyd, A. Heller, S. MacLane, “Samuel Eilenberg (1913–1998)”, Notices of the American Mathematical Society, Vol. 45 (1998), no. 10, 1344–1352.

[5] H. Weyl, Symmetry, Princeton, Princeton University Press, 1952.

[6] Eilenberg–Mac Lane, collected works, Samuel Eilenberg, Saunders Mac Lane, Orlando : Academic Press, 1986.

[7] Algebra, topology, and category theory : a collection of papers in honor of Samuel Eilenberg, edited by Alex Heller, Myles Tierney, New York : Academic Press, [1976]