437. An analysis of those fingerings of the above scale which have not been previously explained, may be found useful.

The fingering of $b''\#$ is of a good type, as it gives the second harmonic (the twelfth) of $e'\#$, improved by the opening of the $a$ hole as a vent-hole. This hole, being placed above its correct position as a note-hole, in order to bring it within reach of the finger, was not very far removed from its correct position as a vent-hole for $b''\#$. This was a favourite fingering of the celebrated Charles Nicholson, and was much in vogue for many years. The $c''\#$ is simply the $b''\#$ sharpened by means of the $b''\#$ hole. The $b''\#$ may be regarded either as a $b''\#$, the harmonic twelfth of $e'\#$, much sharpened by the $c''\#$ hole, or as a $c''\#$, the harmonic twelfth of $f'\#$, assisted by the $c''\#$ hole and much flattened by the closing of the $f'\#$ and $e$ holes. The $d''\#$ is a harmonic twelfth of a true $g'\#$ (with the fingering given for $f'\#$), assisted by the $c''\#$ hole. The $c''\#$ is a harmonic twelfth of $f'\#$, improved by the $c''\#$ hole; this is now the accepted method of forming $d''\#$. The first fingering for $d''\#$ gives the harmonic fifteenth of $d'$, assisted by the $c''\#$ and $f'\#$ holes as vent-holes. The $f''\#$ was fingered according to the only practicable method on such a flute: it may be described as the harmonic fifteenth of $f'\#$, assisted by the opening of the $a$ hole, and flattened by there being no hole for $f$. This fingering gives a wretched note; it has long been discarded in England and in France. The $f''\#$ is like that given by Mersenne. The $g''\#$ is really the same harmonic as the $g''\#$, still further sharpened by the $c''\#$ hole. The $a''\#$ is the fifth harmonic (the nineteenth) of $d'$ assisted by the $e$ and $c''\#$ holes. The note-hole for the last four fingerings is the terminal opening.

438. Quantz gives elaborate directions for the improvement of the imperfect notes of his scale. Sometimes he recommends the turning inwards or outwards of the flute, sometimes the partial closing or unclosing of certain holes for the correction of those defects which he seemed to consider otherwise irremediable. He also lays great stress on the importance of the ear of the performer being adapted for music, and on the necessity for acquiring a knowledge of the proportions of the intervals of the scale. He further says: “It is of great advantage to a flute-player if he know how to make a flute himself, or at least how to tune it.”

439. Gerhard Hoffmann, a famous mathematician and architect born at Rastenburg in 1690, is said by Gerber, who gives Walther (1732) as his authority, to have improved the flute by the addition of a key. Fétis, and also Mendel and Reissman, repeat the statement of Gerber, but a careful search through the work of Walther proves that this author does not even mention