Print your name: ________________________________

- If you need more space to answer a question than we give you, you may use the backs of pages or you may use additional sheets of paper and attach them to the exam. Make sure that we know where to look for your answer!

- Read each question carefully and make sure that you answer everything asked for. Write legibly so that we can read your solutions. Please do not write anything in red.

- We suggest that for solutions that require you to write Java code, you include comments. They will help your grader understand what you intend, which can help you get partial credit.

- Hand in only what you want graded. We do not want your scrap paper unless it contains solutions you want us to grade.

<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Question 1  

24 points

Short answer

(a) (4 points)
In Lab 1 you were asked to implement k-means to reduce the number of colors in a picture to $k$. One way to begin the process was to generate $k$ random colors, and use that color list as the initial color list. When you did that, the final color list usually had many fewer than $k$ colors. Explain what happened, and why it happened so much more often with a random initial color list than when you picked the first $k$ unique colors in the picture as your initial color list.

(b) (4 points)
The factory pattern requires a private constructor and a public “factory” method that is the way to actually create an object of the given class. The classes that implemented the Expression interface followed this pattern, with each having a make or define method. What advantage was gained by doing this, as opposed to just having normal constructors?
(c) (4 points)
The equals method in BinaryTree began:

```java
public boolean equals(Object other) {
    if (other instanceof BinaryTree<?>) {
        BinaryTree<E> t = (BinaryTree<E>) other;
    }
}
```

It would have been much simpler to write it:

```java
public boolean equals(BinaryTree<E> t) {
}
```

and skip all of the silliness of passing an Object and then casting it. What would have gone wrong if we did it the second way?

(d) (4 points)
Exceptions that are not caught are “passed up” the chain of method calls until they reach the main program, and only kill the program if no method in the chain catches the exception. It would be easier to implement exceptions to simply kill the program if the method where the exception occurs does not catch it. What is the advantage of passing the exception back up the call chain instead?
(e) (4 points)
A max-heap data structure has two properties: a shape property and a heap-order property. Explain each, and explain why each property is important.

(f) (4 points)
Your Ellipse class in Lab 3 had a drawShape command that drew an ellipse on a Graphics object by calling drawOval. Calling repaint() when there is an Ellipse object in the drawing somehow causes this drawShape to be called. Describe the steps in this process. You need not remember all of the names of methods and variables (although some are in the Java reference that we passed out). I will also remind you that the instance variable in Editor that referred to the Drawing object was dwg, and it is visible from the inner class CanvasPanel.
Question 2  

20 points

For a short assignment you appended two lists. For this problem you will splice a section out of a linked list and return it as a new list.

Here is part of the code for the SentinelDLLIterator class. Remember, this version has no current element.

```java
public class SentinelDLLIterator<T> implements CS10IteratedList<T> {
    private Element<T> sentinel; // sentinel, serves as head and tail

    private static class Element<T> {
        private T data; // reference to data stored in this element
        private Element<T> next; // reference to next item in list
        private Element<T> previous; // reference to previous item in list

        public Element(T obj) {
            next = previous = null; // no element before or after this one, yet
            data = obj; // OK to copy reference, since obj references an immutable object
        }

        public String toString() {
            return data.toString();
        }
    }

    // Other methods not shown
}
```

```java
/**
 * Constructor for an empty, circular, doubly linked list with a sentinel.
 */
public SentinelDLLIterator() {
    // Allocate the sentinel with a null reference.
    sentinel = new Element<T>(null);
    clear();
}

public void clear() {
    // Make the list be empty by having the sentinel point to itself
    // in both directions.
    sentinel.next = sentinel.previous = sentinel;
}
```

// Other methods not shown
(a) (16 points)
Write a method:

```java
public SentinelDLLIterator<T> splice(Element<T> front, Element<T> rear) {
```

It is to remove the sublist between the Element referred to by `front` and the Element referred to by `rear` from the list stored in this. It should create a new SentinelDLLIterator list consisting of the sublist and return it. The list stored in this should be re-joined, with the Element before `front` preceding the Element following `rear`. So if the original list stored in this consisted of \["a", "b", "c", "d", "e"\] and `front` referred to "b" and `rear` referred to "d", the call to `splice` would return the list \["b", "c", "d"\] and \["a", "e"\] would remain in the list stored in this.

You may assume the following things:

- The list stored in this is not empty.
- `front` refers to an Element that precedes the Element referred to by `rear` in the list stored in this or they both refer to the same element.
- Neither `front` nor `rear` refers to the sentinel of the list stored in this.

Your code must run in constant time. That is, you must manipulate references rather than deleting individual elements from one list and adding them to the other list. Complete the method body.

```java
public SentinelDLLIterator<T> splice(Element<T> front, Element<T> rear) {
```
(b) (4 points)
Give a set of test data for the splice method, consisting of lists and front and rear elements. The first test case is:

List: ["a", "b", "c", "d", "e"]; front: "b"; rear: "d"

Make sure that you test all cases, including boundary cases.
Question 3  10 points

(a)  (5 points)
In class (and in the notes and the book) we saw that repeatedly adding to the end of an ArrayList takes $\Theta(1)$ amortized time if the array inside the ArrayList doubles in size when it becomes full. We saw that by charging 3 tokens per add operation (where a token can pay for adding an item to an empty spot in the current array or copying a single item from the current array to a new array) we always had enough tokens to pay for the operations.

We also said that the Java implementation does not double the size of the array, but rather increases it by half (so the new array is 1.5 times the size of the current array instead of 2 times the size of the current array). We said that the same argument could be adapted to show that by charging 4 tokens per add operation we will always have enough tokens to pay for the add and the copy operations in this case. Explain how this argument works.

(b)  (5 points)
Multiplying two $n \times n$ matrices takes $\Theta(n^3)$ time using the usual algorithm. If it takes .001 second to multiply two $10 \times 10$ matrices, about how long will it take to multiply two $1000 \times 1000$ matrices?
Question 4 16 points

You are hired by the manager of the box office of a theater to help write an automated system for selling tickets. A programmer working for the theater has created the following class to store information about a ticket:

```
public abstract class Ticket {
    private String eventDate; // Date that the event is held

    public Ticket(String date) {
        eventDate = date;
    }

    public abstract double getPrice(); // Returns the price of this ticket

    public String toString() {
        return "Date: " + eventDate + "\nPrice: " + getPrice();
    }
}
```

The manager wants you to create additional classes to represent different types of tickets. In particular, he wants you to create:

- StandingRoom: A category of ticket that costs 10 dollars.
- ReservedSeat: A category of ticket that has a row number and a seat number. Reserved seats in rows 1 to 15 cost 30 dollars and other reserved seats cost 20 dollars.
- StudentTicket: A type of ReservedSeat that costs 5 dollars less than what that reserved seat would normally cost.

In all cases the manager wants you to use good program design, using inheritance where appropriate and not re-writing code unnecessarily. Note that each subclass of Ticket should provide or inherit a toString method that returns the following information about the ticket (appropriately labeled): date, price, and where applicable the row and seat numbers.

While in the real world you would of course define constants to represent things like the prices of the various types of tickets and at which row the price changes for reserved seats, for this exam problem just use the values directly in the program. You would probably also use the Java Date class, but here the date is just a string.
(a) (3 points)
Write the class StandingRoom. Include a constructor that takes the event date. Include any instance variables or methods that you need. Remember all standing room tickets cost 10 dollars.

(b) (8 points)
Write the class ReservedSeat. Include a constructor that takes the event date, the row number, and the column number. Include any instance variables or methods that you need. Remember that reserved seats in rows 1 through 15 cost 30 dollars and other reserved seats cost 20 dollars.
(c) (5 points)
Write the class StudentTicket. Include a constructor that takes the event date, the row number, and the column number. Include any instance variables or methods that you need. Remember that a student ticket costs 5 dollars less than that seat would normally cost. If the price of a normal reserved seat changes, the price of a student ticket should automatically change.
Question 5 15 points

In sa8 you were asked to add a class Negate to the Expression hierarchy. For this problem you are to write the class Square, which implements Expression. One method that you must write is a public make method that follows the factory pattern. It should take an Expression as its only parameter. If given a Constant it should return a Constant whose value is the square of the original constant's value. Given any other Expression it should return a Square object.

Evaluating a Square object should return the square of the value of the Expression given to the make method when the Square object was created. Remember that \( f(x)^2 = f(x) \cdot f(x) \). If the derivative of \( f(x) \) is indicated by \( f'(x) \), then we can define the derivative of the square of \( f(x) \) as:

\[
(f(x)^2)' = 2f(x)f'(x)
\]

The following code in ExpressionDriver:

```java
Variable yVar = define("y", 6.0);
Expression expr = Square.make(minus(Square.make(yVar), constant(4.0)));
System.out.println("The value of the expression:");
System.out.println(expr + " = " + expr.eval());
System.out.println("Its derivative is: " + expr.deriv("y"));
System.out.println("9^2 as simplified in make is: " + Square.make(constant(9.0)));
```

should produce the output:

The value of the expression:
\[((y-2) - 4.0)^2\] = 1024.0
Its derivative is: \((2.0 * ((y-2) - 4.0)) * (2.0 * y)\)
9^2 as simplified in make is: 81.0

The outline of the code for class Square is given on the next page, with places where you should supply code indicated by something like: /** Your code here **/.

For your convenience the code for the Expression interface and the Constant class are given on the page following the outline of the code for Square. Also, the call to create a new Product is:

```java
Product.make(expr1, expr2)
```

where expr1 and expr2 are of type Expression.
public class Square implements Expression {
    /*** Declare instance variable(s) here ***/ 

    private Square(Expression e) {
        /*** Your code here ***/ 
    }

    public double eval() {
        /*** Your code here ***/ 
    }

    public Expression deriv(String v) {
        /*** Your code here ***/ 
    }

    public String toString() {
        /*** Your code here ***/ 
    }

    public static Expression make(Expression e) {
        /*** Your code here ***/ 
    }
}
Code supplied for your reference. Do not write write or change anything here.

```java
public interface Expression {
    /**
     * Evaluates this expression.
     * @return the value of this Expression
     */
    abstract public double eval();

    /**
     * Take the derivative of this expression.
     * @param v the variable with respect to which the derivative is taken
     * @return the derivative of this Expression with respect to the variable v
     */
    abstract public Expression deriv(String v);
}

class Constant implements Expression {
    private double myValue; // the value of the constant

    /**
     * Constructor
     * @param value the value of this Constant
     */
    private Constant(double value) {
        myValue = value;
    }

    /**
     * Creates a constant
     * @param value the value of the constant
     * @return the constant
     */
    public static Constant define(double value) {
        return new Constant(value);
    }

    /**
     * Evaluates this constant
     * @return the value of this constant
     */
    public double eval() {
        return myValue;
    }
}
```
/**
 * Converts this constant to a string
 * @return the string representation of this constant
 */
public String toString()
{
    return "" + myValue; // a sneaky way to convert a double to a String
}

/**
 * Take the derivative of this constant
 * @param v the variable with respect to which the derivative is taken (irrelevant)
 * @return the derivative of this constant, which is 0.0
 */
public Expression deriv(String v)
{
    return new Constant(0.0); // derivative of a constant is 0
}
Question 6  
15 points

In sa9 you filled in method bodies for the class StatesAndCities. The class creates a data structure:

```java
Map<String, Set<String>> stateCityMap = new TreeMap<String, Set<String>>();
```

Here the key to the map was a state name and the value was a set of cities that are in the state. In this form it is easy to find which cities are in a state, but hard to find which states contain a city. To make the second question easy to answer you would want a map where the key is a city name and the value is a set of states that contain the city.

Going from the first form to the second form is called inverting a map. So if the original map were:

MA: {Boston, Concord}
NH: {Concord, Hanover}

then the inverted map would be:

Boston: {MA}
Concord: {MA, NH}
Hanover: {NH}

You are to write a static method that inverts a map. The parameter is the original map. What is returned is the inverted map. The keys in the original map (e.g. the states) become elements of the value sets in the inverted map, and elements of the value sets in the original map (e.g. the cities) become keys in the inverted map. Complete the method begun on the next page.
public static Map<String, Set<String>> invertMap(Map<String, Set<String>> originalMap) {
    // A local variable to accumulate the inverted map, which will be returned.
    Map<String, Set<String>> invertedMap = new TreeMap<String, Set<String>>();
}