Modeling and analysis of active messages in volatile networks*†

C. Okino *(cokino@dartmouth.edu)*
G. Cybenko *(gvc@dartmouth.edu)*
Thayer School of Engineering
Dartmouth College
Hanover, NH 03755

July 1999

Abstract

Interest in and development of mobile agent software systems has burgeoned in the past five years. Code mobility has many attractive attributes for performance and dynamic deployment of new distributed computing and information management applications. An *active message* is a datagram encapsulated as a mobile agent. The agent is persistent in the network, moving from node to node under its own internal routing logic and control at the application layer.

Active messages are particularly attractive in networks that have very unreliable links such as wireless networks in which the nodes are mobile. Such networks experience frequent link failures and other changes in topology. Active messages allow data to propagate between nodes that may never have viable TCP/IP type connections.

In spite of the growing implementation interest in mobile agents and active messaging, there are essentially no analytic models or results dealing with their performance. This paper presents a simple model for active messages in a network with frequent link failures. Using this model, we develop expressions for the expected delivery time of an active message along one path as well as expected delivery time for duplicated messages traversing disjoint paths between source and destination nodes.

1 Introduction

Mobile agents are programs that can migrate from computer to computer under their own control, carrying some part of their execution state with them. Mobile agent systems have been developed for a variety of languages and mobility models [6]. The original motivation for this programming paradigm was to make certain distributed information

*This research was partially supported by National Science Foundation grant CCR-98113744, Air Force Office of Scientific Research Multidisciplinary University Research Initiative Grant F49620-97-1-0382,and DARPA Contract F30602-98-2-0107.

†A copy of postscript may be obtained at http://dream.dartmouth.edu/ cokino.

processing tasks more efficient by moving code *to* the data when the data is large relative to the expected "answer" obtained from processing the data. For example, a unique search of a large, remote database could be efficiently done by moving a relatively small piece of code to the remote database server, executing the code there and sending a relatively small result back to the client machine.

This is essentially the concept behind the well known and successful Java Applet. Mobile agents, however, have the added abilities of making multiple hops within a network and carry some execution state information along with them. Mobile agents are therefore direct generalizations of the applet concept.

Most mobile agents are general programming systems with features added to support program mobility such as state capture, forking, registration, security and network sensing. Mobile agents can therefore easily implement "active messages". An *active message* is data encapsulated as a mobile agent with user-defined routing logic. Mobile agent systems are excessive for implementation of active messages since active messages require only a limited subset of the full mobile agent functionality to perform the routing logic.

Active messages implemented as mobile agents are persistent in a network and can route themselves. Since they are implemented at the applications layer, active messages are not lightweight. Nonetheless, we have found the concept useful in multihop wireless network where nodes are mobile so that disconnects are frequent and routing information may not be reliable.

Consider for example a multihop wireless network with three nodes, A, B and C. Node A and node C are too far apart for a direct wireless link. Node B is mobile and initially close enough to A to be within radio range but is also initially too far from C to establish a radio link. As node B moves from A towards node C, it moves out of the range of Node A and into node C's range. Nodes A and C may never have a TCP-type connection between them. By encapsulating data as an active message from source node A for destination node C, the message can move to node B, remain at node B until node C is reachable and then finally migrate to the destination, node C.

This paper is an initial attempt at modeling and analyzing the performance of such an active messaging system. Clearly, modeling of the movement of all nodes within the system is not possible without a specific mobility model for the nodes which will be very application specific. Our approach is to model nodes that connect sender and receiver as a linear array with unreliable links. The state of a link is a Bernoulli process - at each time interval (which is considered to be relatively long compared to transmission speeds), the link is either up or down according to a Bernoulli probability distribution. If two adjacent links are up simultaneously, the active message can propagate across both and so on. When a link is down, the message "parks" and waits for the next link to be up so it can continue the migration to the ultimate destination.

Section 2 analyzes this simple linear array model with independent Bernoulli links as well as correlated fading channel links. In Section 3, we study the case of multiple paths between sender and receiver. Active messages are duplicated and sent along disjoint paths and each path is modeled as a linear array with identical, independent Bernoulli links. Closed form expressions for expected arrival times are derived for active messages in both cases.

The models and results in this paper are rigorous but preliminary in the sense that much remains to be done. Nonetheless, these results are encouraging in that they indicate that analysis for this type of active messaging model is possible and that much more complexity can be added into future, more realistic models.

2 The Recursive Relationship

Consider a linear network model with $n+1$ nodes as depicted in Figure 1, where the nodes are labelled $a_0, a_1, \cdots, a_n$ and for all $i = 1, 2, \cdots, n$, hop i connects node a_{i-1} to node a_i.

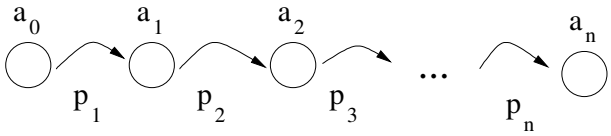


Figure 1: A message traversing multiple hops

We consider a discrete time model, where time is divided into uniform slots. The link failure model for each link is an independent Bernoulli process, where p_i is the probability that hop i is up or available in a given time slot. In Maxemchuk [7], each hop is assumed to have identical average interarrival times as well as identical average service times. In a multi-hop wireless environment with extreme multi-path distortion, this may not be a valid assumption.

Complementary to the link being up or available, we have $1 - p_i$ as the probability that at hop i, the link is down for a given time slot. Thus, the probability that the link is up for the very first time during slot t is equal to the probability that the link was down for all previous $t-1$ time slots.

Under the TCP/IP model, for time slots sufficiently long (e.g. at least as long as the time out period) all links must be up to insure no message loss, so that using the Bernoulli model described above, this occurs with probability $p_1 p_2 \cdots p_n$. The probability that this event occurs for the first time at the s^{th} time slot is $(1 - p_1 p_2 \cdots p_n)^{s-1} p_1 p_2 \cdots p_n$. Thus, the average time to traverse n hops in a TCP/IP model is

$$\begin{aligned} E_{TCP}(n) &= \sum_{s=1}^{\infty} s(1 - p_1 p_2 \cdots p_n)^{s-1} p_1 p_2 \cdots p_n \\ &= \frac{p_1 p_2 \cdots p_n}{(p_1 p_2 \cdots p_n)(1 - 1 + p_1 p_2 \cdots p_n)} \\ &= \frac{1}{p_1 p_2 \cdots p_n} . \end{aligned}$$

The above result is essentially in [1] for end-to-end performance in active networks.

In our model, we consider a *store and forward* methodology allowing a message to move along as many hops as possible in a given time slot (i.e. no propagation delay[1]) without any message loss. That is to say, all links need not be up and available for a message to traverse a link but if all links happen to be available in a given time slot, the message will in fact traverse all hops end-to-end. For example, in the notation of our model, a message can propagate from node a_0 to node a_2 at time slot 1 with probability $p_1 \cdot p_2$, stall there for two time slots since hop 3 is down for the first three time slots with probability $(1 - p_3)^3$ and then propagate to node a_5 at the fourth time slot (with probability $p_3 \cdot p_4 \cdot p_5$). This particular event occurs with probability $p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 (1 - p_3)^3$.

Let $P(n, t)$ be the probability that, under this model, the message first reaches node a_n at time slot t. Recognize that the first message reaches node a_n at time slot t provided

[1] Observing effects due to propagation delay is outside the scope of this paper and is left for future work

the message reached node a_{n-1} at time slot $s \leq t$, stalled for $t-s$ time slots and then traversed hop n to reach node a_n at time slot t. The probability of this event is $P(n-1,s)(1-p_n)^{t-s}p_n$, which also includes the case $t = s$. Since these events are independent and disjoint, we have

$$P(n,t) = \sum_{s=1}^{t} P(n-1,s)(1-p_n)^{t-s}p_n \ , \tag{1}$$

and

$$E_1(n) = \sum_{t=1}^{\infty} tP(n,t) \ . \tag{2}$$

Theorem 1 *The expected time for a message required to traverse n hops in the model is* $E_1(n) = \sum_{j=1}^{n} \left(\frac{1}{p_j}\right) - n + 1$.

Sketch Proof of Theorem 1. Using (1), it can be shown that

$$P(n,t) = p_n P(n-1,t) + (1-p_n)P(n,t-1) \ . \tag{3}$$

Using (2) and then (3) we can separate $P(n,t)$ into a sum of terms that is the average delay at the $n-1^{st}$ hop and the n^{th} hop as shown below:

$$E_1(n) = p_n E_1(n-1) + (1-p_n)[E_1(n)+1] \ .$$

Re-writing the result above, we have the average delay equal to $\sum_{j=2}^{n} \left(\frac{1-p_j}{p_j}\right) + E_1(1)$. Recognizing that $E_1(1) = E_{TCP}(1) = \frac{1}{p_1}$, we are done.

□

Corollary 2 *If at each hop, the probability that the link is up is equal to p, then* $E_1(n) = n\frac{1-p}{p} + 1$.

Proof of Corollary 2. Use Theorem 1 with $p_i = p$ for all $i = 1, 2, \cdots, n$.

□

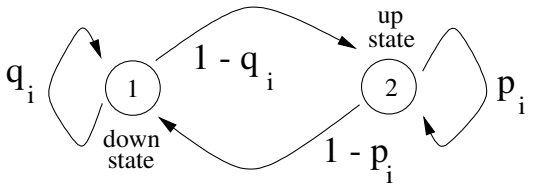
Intuitively, when all the link probabilities are equal to p, we could have obtained an upper bound average delay by using the single hop result of $\frac{1}{p}$ and multiplying this by n. This is in the spirit of the results in [7] for exponentially distributed interarrival and service times where a message may be restricted from traversing multiple hops in a single time slot. Moreover, Maxemchuk's use of the Kleinrock model [5] is perhaps too restrictive by modeling $M/M/1$ queues which do not translate well for modeling the distribution of multipath channel distortion.

2.1 The Correlated Channel

In this section, we consider the case where all hops are modeled as passing through a correlated fading channel. Specifically, we consider the multipath fading of a wireless mobile channel modeled as a Raleigh distribution. We can then view the success and failure of a message being transmitted over a hop approximately as a simple two-state Markov chain [4] as depicted in Figure 2 (a). The concept of modeling success and failures of a data block for a correlated Raleigh fading channel was first considered by

[8] and further developed for TCP wireless fading channels in [3]. We shall consider the correlation effects of a link being available during different time slots. Note that in this subsection, the definition of p and p_i has been altered to suite the stochastic nature of the fading channel model.

For each time slot, the probability that a link is up and available will depend on whether the link was available or not available during the previous time slot. Specifically, for hop i, the transition probability matrix is shown in Figure 2 (b), where p_i is the probability that hop i is up during time slot s given that hop i was up during time slot $s-1$ (i.e. in Figure 2 (a), being in state 2 during time slot $s-1$ and remaining in state 2 for the next state, s). Similarly, $1-q_i$ is the probability that hop i is up during time



(a) Two state markov chain

$$M_i = \begin{pmatrix} p_i & 1-p_i \\ 1-q_i & q_i \end{pmatrix}$$

(b) Transition matrix

Figure 2: A simple markov chain

slot s given that hop i was down during time slot $s-1$ (i.e. in Figure 2 this is equivalent to being in state 1 during time slot $s-1$ and remaining in state 1 for the next state, s). Given the channel transition matrix for each hop i we can easily obtain the steady state probability of the hop being up, p_i' and similarly, the steady state probability of the hop being down, $1-p_i'$ for each hop i. i.e. For each hop i we have $p_i' = \frac{1-q_i}{2-p_i-q_i}$ and $1-p_i' = \frac{1-p_i}{2-p_i-q_i}$.

Let $P'(n,t)$ be the probability that, under this model, the message first reaches node a_n at time slot t. Recognize that a message arriving at hop $n-1$ for the first time at exactly time slot s must always be in state 2 at time slot s. Moreover, recognize that the message reaches node a_n at time slot t for the first time provided the message reached node a_{n-1} at time slot $s \leq t$, stalled for $t-s$ time slots and then traversed hop n to reach node a_n at time slot t. The probability of this event is $P'(n-1,s)(1-p_n)q_n^{t-s-1}(1-q_n)$, for $s < t$, and $P'(n-1,s)p_n$, for $s = t$. Since these events are independent and disjoint, we have

$$P'(n,t) \;=\; P'(n-1,t)p_n + \sum_{s=1}^{t-1} P'(n-1,s)(1-p_n)q_n^{t-s-1}(1-q_n) \; . \qquad (4)$$

The method of deriving the n^{th} hop average time is similar to the recursive method of proof for Theorem 1 and is left to the reader.

Theorem 3 *The expected time for a message required to traverse n hops in the model is*

$$E_1(n) = 1 + \frac{1-p_1}{(1-q_1)(2-p_1-q_1)} + \sum_{j=2}^{n} \frac{1-p_j}{1-q_j} \; .$$

By using Theorem 3, when the transition matrix is equal for each hop, i.e. for all hops i, $p_i = p$ and $q_i = q$, we have the average delay at the n^{th} hop equal to $1 + \frac{1-p}{(1-q)(2-p-q)} + \frac{(n-1)(1-p)}{1-q}$. Recognize that setting $q_i = 1-p_i$ for all i, we get $p_i' = p_i$ and Theorem 3 is equivalent to the result of Theorem 1 as expected.

3 The "Binomial" Representation

In this section, we consider the case when the link failure at each hop is the same. As in Corollary 2, we assume that for each link, the probability that the link is up and available is p.

Since $P(n,t)$ is the probability that the message reaches the n^{th} hop for the first time at exactly time slot t, recognize that the expression for $P(n,t)$ is simply $(1-p)^{t-1}p^n$ multiplied by all the possible ways to stall $t-1$ times in n possible hops. i.e.

$$P(n,t) = \binom{n-1+t-1}{t-1}(1-p)^{t-1}p^n . \quad (5)$$

This is the well known binomial distribution.

An alternate derivation for Corollary 2 can be shown using the following useful proposition.

Proposition 4 *For* $F(n) = \sum_{l=0}^{\infty}\binom{n+l}{l}q^l$, *we have* $F(n) = \frac{1}{(1-q)^{n+1}}$.

The derivation for the above proposition is straight forward using a binomial identity and is left to the reader.

The concept of transmitting identical copies of a message along disjoint paths is characterized by Maxemchuk as redundant dispersity routing [7], in the form of packet transmissions. We now consider the case where sender (source node) may "spawn" identical messages in an attempt to reduce the average message delay. Specifically, the source is allowed to duplicate M identical messages to be transmitted to the destination along n disjoint hops. In this model, we assume that each message will traverse the same number of hops n. Moreover, for any link, the probability that the link is up and available at a given time slot is assumed to be equal to p for each time slot.

Let $P(n,s)$ be the probability of a message reaching hop n first at time slot s and p be the probability that the link is up and available at hop n in a given time slot. We can express the probability of a message reaching hop n first at time slot s in terms of a sum of the probabilities of a message reaching hop $n-1$ first at previous time slots $l \leq s$. i.e. $P(n,s) = \sum_{l=1}^{s} P(n-1,l)(1-p)^{s-l}p$. The probability of a message reaching hop n after time slot t can be expressed by one minus the sum of all the probabilities of the message arriving before or during time slot t. Let $Q(n,t)$ be the probability that the message arrives at hop n after time slot t. i.e. $Q(n,t) = \sum_{s=t+1}^{\infty} P(n,s) = 1 - \sum_{s=1}^{t} P(n,s)$. For the case of M identical messages transmitted along n hops, the probability that at least one of the messages reaches hop n first at time slot t, $P_M(n,t)$ can be expressed by $\sum_{i=1}^{M}\binom{M}{i}P(n,t)^i Q(n,t)^{M-i}$. Thus, the average time for *at least* one of the M messages to reach hop n can be expressed by

$$E_M(n) = \sum_{t=1}^{\infty} t \sum_{i=1}^{M} \binom{M}{i} P(n,t)^i Q(n,t)^{M-i} . \quad (6)$$

Note that when $M = 1$ we get (2).

3.1 The M identical message analysis

Before stating the theorem we introduce the following notation. Let

$$G(M,n,q_1,q_2,\ldots,q_M) = \frac{p^{Mn}(q_1 q_2 \cdots q_M)^{n-1}}{\{(n-1)!\}^M (1-q_1 q_2 \cdots q_M)\prod_{i=1}^{M}(1-q_i)} .$$

Theorem 5 *For a network with each link probability p, the average time to traverse n hops using M identical messages is the $n-1^{st}$ partial derivative of $G(M,n,q_1,q_2,\ldots,q_M)$ with respect to each of the q_i where $i=1,2,\ldots,M$ and then evaluated at $q_i=1-p$. i.e.*

$$E_M(n)=\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}G(M,n,q_1,q_2,\cdots,q_M)\Big|_{q_1=q_2=\cdots=q_M=1-p}.$$

Proof of Theorem 5. Recognize that

$$\begin{aligned}P_M(n,t) &= \sum_{i=1}^{M}\binom{M}{i}P(n,t)^iQ(n,t)^{M-i}\\ &= (P(n,t)+Q(n,t))^M-Q(n,t)^M\ .\end{aligned}\tag{7}$$

Using (6) and (7), we have

$$E_M(n) = \sum_{t=1}^{\infty}t\{(P(n,t)+Q(n,t))^M-Q(n,t)^M\}\ .\tag{8}$$

Let $E^{PQ}(n)$ be equal to the first term on the right hand side of (8) and $E^{Q}(n)$ be equal to the second term on the right hand side of (8). i.e.

$$E^{PQ}(n)=\sum_{t=1}^{\infty}t(P(n,t)+Q(n,t))^M\ ,$$

$$E^{Q}(n)=\sum_{t=1}^{\infty}tQ(n,t)^M\ ,$$

and

$$E_M(n) = E^{PQ}(n)-E^{Q}(n)\ .\tag{9}$$

Then for $E^{PQ}(n)$ where $v=q_1q_2\cdots q_M$ and $q_1,q_2,\ldots,q_M$ are all evaluated at q, we have

$$\begin{aligned}E^{PQ}(n) &= \sum_{t=1}^{\infty}t(P(n,t)+Q(n,t))^M\\ &= \sum_{t=1}^{\infty}t\{P(n,t)+\sum_{s=t+1}^{\infty}P(n,s)\}^M\\ &= \sum_{t=1}^{\infty}t\{\sum_{s=t}^{\infty}P(n,s)\}^M\\ &= p^{Mn}\sum_{t=1}^{\infty}t\{\sum_{s=t}^{\infty}\binom{n-1+s-1}{s-1}q^{s-1}\}^M\\ &= p^{Mn}\sum_{t=1}^{\infty}t\{\frac{1}{(n-1)!}\frac{\partial^{n-1}}{\partial q^{n-1}}\sum_{s=t}^{\infty}q^{n-1+s-1}\}^M\\ &= p^{Mn}\sum_{t=1}^{\infty}t\{\frac{1}{(n-1)!}\frac{\partial^{n-1}}{\partial q^{n-1}}\frac{q^{n-1+t-1}}{1-q}\}^M\end{aligned}$$

$$
\begin{aligned}
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\left\{\prod_{i=1}^{M}\frac{1}{1-q_i}\right\}\sum_{t=1}^{\infty}tv^{n-1+t-1}\\
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\left\{\prod_{i=1}^{M}\frac{1}{1-q_i}\right\}v^{n-1}\frac{\partial}{\partial v}\sum_{l=0}^{\infty}v^{l+1}\\
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\left\{\prod_{i=1}^{M}\frac{1}{1-q_i}\right\}v^{n-1}\frac{\partial}{\partial v}\frac{v}{1-v}\\
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\left\{\prod_{i=1}^{M}\frac{1}{1-q_i}\right\}\frac{v^{n-1}}{(1-v)^2}\\
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\frac{(q_1q_2\cdots q_M)^{n-1}}{(1-q_1q_2\cdots q_M)^2\prod_{i=1}^{M}1-q_i}\ . \qquad (10)
\end{aligned}
$$

Recognize that $E^Q(n)$ is simply one $E^{PQ}(n)$ with out the last term, so that

$$
\begin{aligned}
E^Q(n) &= \sum_{t=1}^{\infty}t\{\sum_{s=t+1}^{\infty}P(n,s)\}^M\\
&= p^{Mn}\sum_{t=1}^{\infty}t\{\frac{1}{(n-1)!}\frac{\partial^{n-1}}{\partial q^{n-1}}\frac{q^{n-1+t}}{1-q}\}^M\\
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\frac{(q_1q_2\cdots q_M)^{n}}{(1-q_1q_2\cdots q_M)^2\prod_{i=1}^{M}1-q_i}\ . \qquad (11)
\end{aligned}
$$

Combining (9), (10), and (11), we have

$$
\begin{aligned}
E_M(n) &= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\frac{(q_1q_2\cdots q_M)^{n-1}(1-q_1q_2\cdots q_M)}{(1-q_1q_2\cdots q_M)^2\prod_{i=1}^{M}1-q_i}\\
&= \frac{p^{Mn}}{\{(n-1)!\}^M}\frac{\partial^{n-1}}{\partial q_1^{n-1}}\frac{\partial^{n-1}}{\partial q_2^{n-1}}\cdots\frac{\partial^{n-1}}{\partial q_M^{n-1}}\frac{(q_1q_2\cdots q_M)^{n-1}}{(1-q_1q_2\cdots q_M)\prod_{i=1}^{M}1-q_i}\ . \qquad (12)
\end{aligned}
$$

□

Corollary 6 *For a single hop(i.e. $n=1$) we have $E_M(1)=\frac{1}{1-(1-p)^M}$.*

Recognize that for $M=1$ in the above corollary, we get the expected result of $\frac{1}{p}$ for the average time for a single message to traverse a single hop. In this particular case, the average delay is identical to the average delay for a TCP connection described earlier. We now consider the other extreme case.

Corollary 7 *As the number M of identical messages increases, the average time to traverse a single hop attains the information theoretic bound of 1. i.e. $\lim_{M\to\infty}E_M(1)=1$.*

As a consequence of Theorem 5, for M identical messages traversing n hops, $Mx(n-1)$ partial derivatives need to be calculated. Specifically, for the case of two identical messages($M=2$), $2n-2$ partial derivatives need to be computed. This motivates the following result, originally considered by Chiou and Li [2] as the "two-copy" case, but for packets transmitted through an $M/M/1$ queue.

Theorem 8 *Suppose that for each hop the probability that the hop being up is equal to p. Then for a two identical message model, the average time for at least one message to traverse n hops is*

$$E_2(n) = \frac{\partial^{n-1}}{\partial q_2^{n-1}} G_2(n, q_1, q_2)\Big|_{q_1=q_2=1-p},$$

where

$$G_2(n, q_1, q_2) = \frac{p^{2n}}{(n-1)!} \frac{q_2^{n-1}(1+q_2)(1+(n-1)q_1q_2)}{(1-q_2)(1-q_1q_2)^{n+1}} .$$

Recognize that for two identical messages, from Theorem 8, the average delay over n hops requires calculating $n-1$ derivatives, half the number required using Theorem 5.

3.2 Simulations

Suppose T_i is the total time for the i^{th} message to traverse n hops, where $i = 1, 2, \ldots, M$. In the simulation where M identical messages are considered, we compute the time required for each of the M parallel disjoint paths and choose the minimum among these times for each time slot. i.e. For each time sample we choose $T = \min_{i=1,2,\ldots,M}\{T_i\}$.

The simulated average times were all computed for approximately 10000 iterations. Specifically, each message end-to-end time computation was averaged over 10000 samples. Figure 3(a) displayed the theoretical values for TCP versus a single message using our store and forward model. These results imply that for very unreliable links, there is a significant gain in end-to-end delay performance over multiple hops utilizing our store and forward model. Conversely, for extremely reliable links (i.e. $p = .9$ in Figure 3 (a)), there is very little motivation to consider this model versus TCP. Figures 3(b-c)

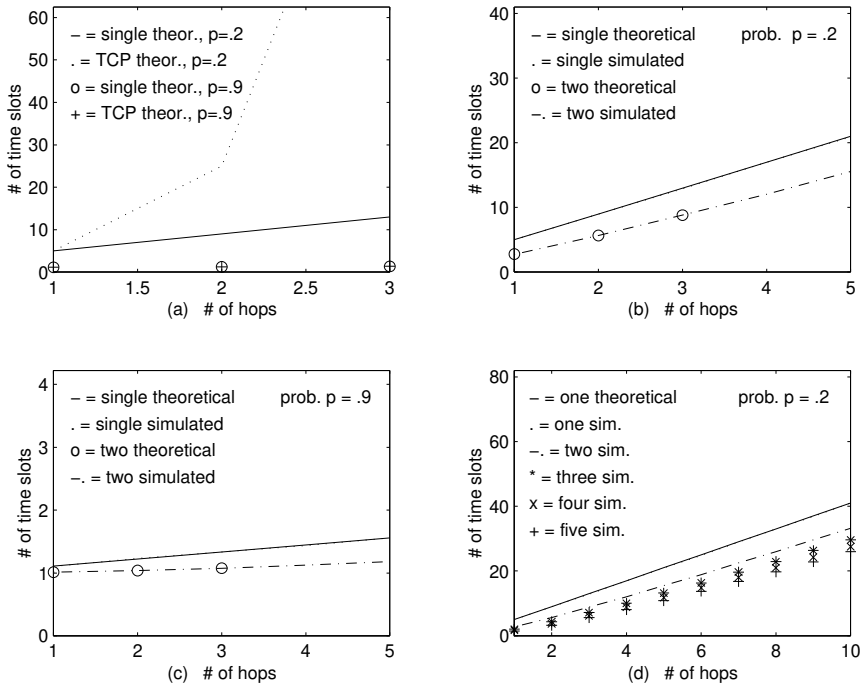


Figure 3: Theoretical and simulation plots for multiple messages

displayed the theoretical values for a single message and two identical message case as well as the simulated average end-to-end delay for link probabilities $p = 0.2, 0.9$ respectively. Note that theoretical values for the two identical message case are computed only for the first three hops. As expected, the two identical messages case reduces the average delay relative to the single message case. However, for larger link probabilities, the improvement may be considered negligible coinciding with the previous results for TCP. This is reaffirmed by the simulation results depicted in Figure 3(d) where we examine up to five identical messages on disjoint paths for link probabilities $p = 0.2$. Moreover, Figure 3(d) suggest that the amount of improvement by replicating additional identical messages dramatically reduces.

4 Brief Closing Remarks

In this paper, we considered two methodologies for obtaining closed form solutions for average active message delay incurred over multiple hops in a network. The recursive relationship methodology proved useful in obtaining a closed form solution over multiple hops with correlated link failure probabilities. Using the binomial relationship, we obtained a closed form solution for average time of a message to traverse n hops as a function of the number of duplicate copy messages transmitted. Although these result suggest a potential gain for networks with low link probabilities, there is clearly a trade-off we need to investigate for buffer allocations and added link capacity utilization.

References

[1] S. Bhattacharjee, K. L. Calvert, and E. Zegura. Active netowrking and the end-to-end arguement. In *Proc. of International Conference on Network Protocols 1997*, Oct 1997.

[2] S-N. Chiou and V.O.K. Li. An optimal two-copy routing scheme in a communication network. In *IEEE Proceeding INFOCOM '88*, pages 288–97, Apr 1988.

[3] A. Chockalingam, Michele Zorzi, and Ramesh R. Rao. Performance of tcp on wireless fading links with memory. In *Proc. IEEE ICC'98*, Atlanta, GA, Jun 1998.

[4] E.N. Gilbert. Capacity of a burst-noise channel. *The Bell System Technical Journal*, 39:1253–1265, Sep 1960.

[5] L. Kleinrock. *Communication Nets: Stochastic Message Flow and Delay.* New York: McGraw Hill, 1964.

[6] Matthias Klusch. *Intelligent Information Agents - Agent-Based Information Discovery and Management on the Internet*, chapter "Mobile Agents for Distributed Information" by B. Brewington, R. Gray, K. Moizumi, D. Kotz, G. Cybenko, and D. Rus. Springer, 1998.

[7] N.F. Maxemchuk. Dispersity routing. In *IEEE Proceeding ICC '75*, pages 41–10–41–13, Jun 1975.

[8] M. Zorzi, R. R. Rao, and L. B. Milstein. On the accuracy of a first-order markov model for data transmission on fading channels. In *Proc. IEEE ICUPC'95*, pages 211–215, Tokyo, Japan, Nov 1995.