CS 109
Spring 2008
Theory of Computation: Advanced

Homework 5
Due Tue Apr 22, 5:00pm

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General Instructions: Same as in Homework 1.
Honor Principle: Same as in Homework 1.

10. For a string $x \in \{0,1\}^*$, let $N_1(x)$ denote the number of 1s in x. The *majority* function $\text{MAJ}_n : \{0,1\}^n \to \{0,1\}$ is defined as follows:
$$\text{MAJ}_n(x) = \begin{cases} 1, & \text{if } N_1(x) \geq n/2, \\ 0, & \text{otherwise.} \end{cases}$$

Show that MAJ_n can be computing using $O(n)$-sized circuits. [This is essentially Sipser's Problem 9.26 — if you use the approach suggested in the book, you need to first solve (in sufficient detail) any subproblems that come up, such as Sipser's Problem 9.24.]

(Just one problem this time. Maybe more after we've seen more of circuit complexity.)