General Instructions: Same as in Homework 1.

Honor Principle: For this homework, you should work entirely on your own and not discuss with anyone.

15. Give a full formal proof that \( \text{ZPP} = \text{RP} \cap \text{coRP} \). [2 points]

16. For constants \( 0 < \alpha < \beta < 1 \), define the class \( \text{BPP}_{\alpha,\beta} \) to be the class of all languages \( L \subseteq \Sigma^* \) such that there exists a PTM \( M \) that runs in polynomial time and behaves as follows on an input \( x \in \Sigma^* \):

\[
\begin{align*}
    x \notin L & \Rightarrow \Pr_{R}[M(x,r) = 1] \leq \alpha, \\
    x \in L & \Rightarrow \Pr_{R}[M(x,r) = 1] \geq \beta.
\end{align*}
\]

Note that our definition of BPP in class coincides with \( \text{BPP}^{\frac{1}{3}, \frac{2}{3}} \) in this notation.

Using Chernoff bounds, give a full formal proof that for all \( \alpha \) and \( \beta \) as above, \( \text{BPP}_{\alpha,\beta} = \text{BPP} \). [2 points]

Recall that the Chernoff bound we saw in class had the following general form. Let \( \{X_1, \ldots, X_n\} \) be independent indicator random variables with \( \mathbb{E}[X_i] = p_i \). Suppose \( X = \sum_{i=1}^{n} X_i \) and let \( p \) be such that \( np = p_1 + \cdots + p_n \). Then, for any \( \delta > 0 \):

\[
\Pr[X \geq (1 + \delta)np] \leq \left( \frac{e^\delta}{(1 + \delta)^{1+\delta}} \right)^{np}.
\]

We also have a similar inequality bounding deviations of \( X \) below its mean. For \( 0 < \delta < 1 \):

\[
\Pr[X \leq (1 - \delta)np] \leq \left( \frac{e^{-\delta}}{(1 - \delta)^{1-\delta}} \right)^{np}.
\]