26. We defined the classes AM and MA, involving two players Arthur (a probabilistic polynomial-time verifier) and Merlin (a computationally unbounded “magical” prover who can observe Arthur’s coin tosses). A formal definition of AM can be given as follows.

Suppose $L \subseteq \{0, 1\}^*$ is a language, and Merlin is trying to prove to Arthur that $x \in L$. Let $V(x, r, w) \in \{0, 1\}$ denote the outcome of Arthur’s verifier ($1 = \text{ACCEPT}, 0 = \text{REJECT}$) on input $x \in \{0, 1\}^*$, random string $r \in \{0, 1\}^*$ and witness $w \in \{0, 1\}^*$ provided by Merlin. We say $L \in \text{AM}$ if $\exists V$ such that

$$x \notin L \implies \Pr_r[\exists w \ (V(x, r, w) = 1)] \leq \frac{1}{3},$$

$$x \in L \implies \Pr_r[\exists w \ (V(x, r, w) = 1)] \geq \frac{2}{3}.$$ 

Notice that the definition captures the nature of the interaction: Merlin provides a witness $w$ based on both $x$ and $r$. The class MA differs from this in that Merlin must provide a witness $w$ in advance, and Arthur then uses a random string $r$ to do the verification. Give an analogous formal definition of MA; use the specific constants $\frac{1}{3}$ and $\frac{2}{3}$ as above. Then, give a full formal proof that $\text{MA} \subseteq \text{AM}$. [2 points]

27. Prove that $\text{AM} \subseteq \text{PH}$. Try to find the lowest level you can within the polynomial hierarchy that contains $\text{AM}$. [2 points]