Welcome to CS 39
Theory of Computation

Professor Amit Chakrabarti
Teaching Assistant: Vibhor Bhatt
http://www.cs.dartmouth.edu/~cs39

Administrivia: Lectures
- We will use every X-hour.
- It is vital to attend every lecture, or you’ll be overwhelmed trying to catch up.
- Almost every lecture will define new concepts
  - Pay attention!
  - Stop me the moment you don’t follow something
- Lectures will not slavishly follow textbook
  - Take good notes in class
  - Will be helpful during exams, and later in life

Administrivia: Grading
- HW: 35, Quizzes: 15, Midterm: 20, Final: 30
- Homework given out Wed, due next Wed
  - Homework due at start of class (late policy: see web)
  - Will require plenty of “think time”
  - Start early!
- Two in-class quizzes
- Take-home midterm, take-home final

Administrivia: Honor Code
- Homework
  - You can work on homework problems in groups.
  - But when you start writing your solutions you must work alone and write in your own words.
  - If you borrowed a key idea for a solution from your classmate(s), acknowledge them like a true scholar.
- Exams
  - You may not discuss exam problems with anybody, not even with Vibhor or me.
Administrivia

• If you need card access to Sudikoff on evenings (after 6pm) and weekends, see Kelly Clark.

• If you have a documented disability and need academic adjustments, please see me privately before Oct 10.

A basic theme of CS 39

• “Computers cannot solve everything.”
• Perhaps this seems obvious to you, as stated:
  – How do I become a millionaire?
  – How do I unite quantum theory and gravitation?
  – How do I find out if God exists?
• These “problems” are not well-posed.
  – Or, these are not “computational problems.”
• A well-posed problem should describe the desired output for every possible input.

What CS 39 is about

• What can or cannot be computed?
  – What does “to compute” mean?
  – What is a “computer”?
  – Are there really things that cannot be computed?
• How quickly can something be computed?
  – Will you give me the latest snazziest computer?
• Abstract models of computation

What CS 39 is about

• Finite automata
  – Regular languages
• Push-down automata
  – Context-free languages
• Turing machines
  – Decidable languages (and undecidable ones)
• Polynomial time bounded Turing machines
  – P, NP, NP-complete
Okay, what CS 39 is really about

Proofs, proofs, proofs, proofs
And…
More proofs!

Relax

Although this is a mathematical course…
• No derivatives or integrals
• No binomial coefficients
• No number-crunching calculations
• No probability theory

Just mathematical notation
and the power of reasoning.

Infinite Loop Tester

• You are a grader for CS 5
• Students submit a program “foo.java”
  – You have test input files “1.inp”… “100.inp”
  – Every test case correctly handled: 1 point
  – Infinite loop: -20 points (penalty)
• You wish to automate the grading process
  – Write a program “ILT.java” which takes two input
    files ---“foo.java” and “i.inp” --- and checks whether
    “foo.java” enters an infinite loop on input “i.inp”

It’s impossible

• The Infinite Loop Tester cannot be written!

• This is not because of any deficiency or peculiarity of the Java programming language, the task simply cannot be done by a computer.

• Truly remarkable that a well-posed computational problem cannot be solved by a computer!
Hilbert’s Tenth Problem

• Given a system of polynomial equations with integer coefficients, like
  \[ \begin{align*}
  x^2 - 3xz &= 5 \\
  7xy + 4y^3 - xz^2 &= 2
  \end{align*} \]
• Is there a solution in the integers?
• i.e., in this case, are there integers \( x, y, z \) that satisfy both these equations?

It’s impossible

• A computer program that solves Hilbert’s Tenth Problem cannot be written.
• Again, it does not matter what type of computer or what programming language you use.
• Proof beyond the scope of this course.
  (But contact me later if you’re curious.)

A Goal

One of the main goals of CS 39 is to develop a theory that enables us to prove such results.

Math Notation: Sets

• \( A = \{1,2,3,4,5\} \quad B = \{1,3,5,7,9\} \)
• \( |A| = 5 \)
• \( A \cup B = \{1,2,3,4,5,7,9\} \)
• \( A \cap B = \{1,3,5\} \)
• \( A \times B = \{(1,1), (1,3), (1,5), \ldots, (5,5), (5,7), (5,9)\} \)
• \( A - B = \{2,4\} \)
• \( 2^A = \{\emptyset, \{1\}, \{2\}, \ldots, \{1,2\}, \{1,3\}, \ldots, \{1,2,3,4,5\}\} \)
More Math Notation

- A = {1,2,3,4,5}  B = {1,3,5,7,9}
- \{x \in A \cup B : x \mod 3 = 1\} = \{1,4,7\}
- For integer n, let S(n) = \{n, n^2\}
- S(9) \cup S(10) = \{9,81,10,100\} = \{9,10,81,100\}
- \bigcup_{i=1}^{3} S(i) = \{1,1\} \cup \{2,4\} \cup \{3,9\} = \{1,2,3,4,9\}
- \bigcup_{x \in B} S(x) = \{1,3,5,7,9,25,49,81\}
- \{S(n) : n \in A\} = \{\{1\},\{2,4\},\{3,9\},\{4,16\},\{5,25\}\}