When describing a Turing Machine, be sure to state explicitly what “extra features” (such as multiple tapes and/or nondeterminism) you are planning to use, if any. Feel free to use implementation descriptions, as opposed to formal descriptions, unless explicitly asked to do otherwise. If asked to give formal descriptions, draw the transition diagram rather than writing it out as a table.

As usual, please think carefully about how you are going to organize your answers before you begin writing. Make sure your answers are complete, clean, concise and rigorous.

1. Formally describe a single-tape deterministic TM that decides the language \( \{w \in \{0,1\}^*: w = w^R\} \). In addition to drawing the TM’s transition diagram, to get any credit, you must (1) explain the overall strategy clearly in English, and (2) describe what each state of your machine stands for. (Without such explanations and comments, grading would be impossible.) [20 points]

2. Formally describe a two-tape deterministic TM that decides the language \( \{w \in \{0,1\}^*: w = w^R\} \). Provide a diagram, plus explanations and comments, exactly as above. You may assume that in a particular move one (or both) of the heads is allowed to remain stationary. Thus, the transition function for such a TM would look like

\[
\delta : Q \times \Gamma^2 \rightarrow Q \times \Gamma^2 \times \{L, R, S\}.
\]

When drawing a diagram, if \(\delta(q, a, b) = (r, c, d, L, S)\), for example, you would draw an arrow from \(q\) to \(r\) and label it “\((a, b) \rightarrow (c, d), (L, S)\)”.

Appreciate the ease of programming with two tapes for this language. [15 points]

3. Formally describe a two-tape NDTM for the language \( \{ww : w \in \{0,1\}^*\} \). Again, provide a diagram, plus explanations and comments.

Appreciate the ease of programming resulting from nondeterminism and the availability of two tapes. [20 points]

4. Prove that decidable languages are closed under (a) union, (b) intersection, (c) complement, (d) concatenation, and (e) Kleene star. [15 points]

5. Prove that recognizable languages are closed under (a) union, (b) intersection, and (c) concatenation. [15 points]

They are also closed under Kleene star. I’m sure you can mentally toss off a proof of this fact. Smile smugly as you figure this out (no need to turn it in).

6. Show that a \(k\)-tape TM \(M\) can be simulated by a single-tape TM \(M'\) in such a way that a computation which takes time \(t\) (i.e., \(t\) steps of one configuration yielding another) on \(M\) takes time \(O(t^2)\) on \(M'\). The big-\(O\) notation may hide a constant that depends on \(k\). You may assume that \(t \geq |x|\), where \(x\) is the input to \(M\).

While not strictly necessary, it might help you to use a slightly different multtape-to-single-tape encoding from the one described in class, such as one that interleaves the symbols on the \(k\) tapes. [15 points]
Challenge Problems

**CP6:** Prove that every context-free subset of $0^*$ is regular.

**CP7:** Make an appropriate formal definition of a *deterministic* pushdown automaton with two stacks. Call such an automaton a 2-S-DPDA. Prove, by formal construction, that every decidable language can be accepted by a 2-S-DPDA.