Homework 3
Due 2017-Jan-30, 11:59pm

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General Instructions. Same as for Homework 1.

Honor Prinicple. Same as for Homework 1.

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6. Let G be a group of permutations of $[n]$ (not necessarily the group of *all* permutations). Let $\alpha \in \{0,1,\star\}^n$ be a partial assignment. For $\pi \in G$, define $\alpha \circ \pi$ to be the partial assignment obtained by applying the permutation π to α. We define a function $F_{\alpha,G} : \{0,1\}^n \to \{0,1\}$ as follows:

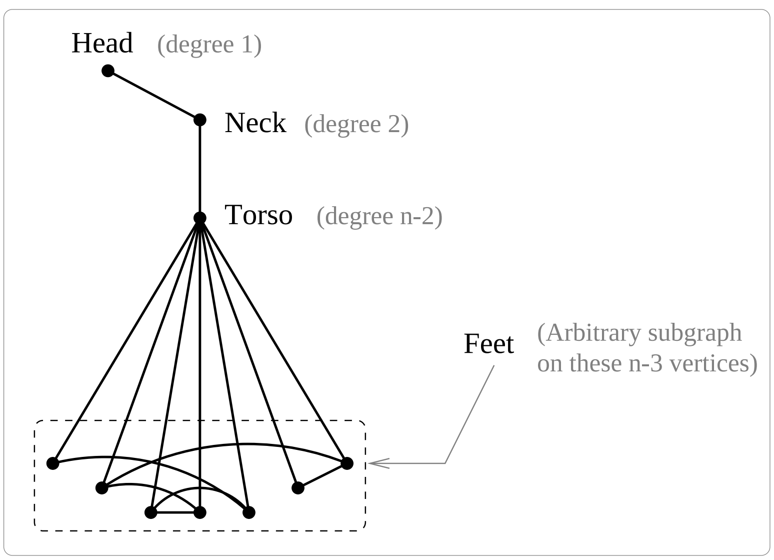
$$\forall\, x \in \{0,1\}^n : \quad F_{\alpha,G}(x) = \begin{cases} 1, & \text{if } \exists\, \pi \in G : \ x \text{ matches } \alpha \circ \pi \\ 0, & \text{otherwise.} \end{cases}$$

   6.1. Prove that $\mathrm{C}^1(F_{\alpha,G}) = |\operatorname{Ex}(\alpha)|$. Recall that $\operatorname{Ex}(\alpha)$ denotes the set of indices of exposed bits in α.

   6.2. Prove that $s(F_{\alpha,G}) \ge |\operatorname{Ex}(\alpha)|/2$.

7. Even while working on lower bounds one often has to prove upper bounds, if only to provide counterexamples to plausible but false lower bound conjectures. In the early 1970s it was conjectured that *every* nontrivial graph property f_n on n-vertex graphs has $\mathrm{D}(f_n) = \Omega(n^2)$. We will soon prove this for *monotone* f_n (this is the Rivest-Vuillemin Theorem), but what about non-monotone properties?

   Call an n-vertex graph a *scorpion* if it has the structure shown in the following figure.



   Let $\textsc{scorp}_n$ be the property of being a scorpion.

   7.1. Show that $\textsc{scorp}_n$ is not monotone.

   7.2. By designing a suitable query strategy (i.e., an algorithm) that queries at most $6n$ of the $\binom{n}{2}$ Boolean variables representing the possible edges of an n-vertex graph, show that $\mathrm{D}(\textsc{scorp}_n) \le 6n = O(n)$.

   Hint: If an input graph is indeed a scorpion, it is easy to verify this if an oracle tells you which vertex is the torso.