Animating Sand as a Surface Flow

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Abstract

This paper presents a new efficient method for animating sand and other granular materials in 3D scenes. Our method couples 2D and 3D simulation techniques in a physically based way. A surface flow model of granular material-the BCRE model-is used to separate sand piles into two layers: a surface flowing layer and a static layer. The surface layer is simulated using discrete element method (DEM) to capture the detailed flowing behavior, while the invisible and static layer is represented by a height field for efficiency. The matter transfer between the two layers is modeled based on the surface flow equations through a particle interface. We demonstrate that our method leads to significant improvements of computational efficiency compared to standard discrete element method, without sacrificing the rich 3D animation effects.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Physically based modeling

1. Introduction

Sand, with all granular materials, has unique nature properties. It can flow under external forces as a fluid and stabilize into sand piles as a solid. Animating scenes with sand in computer graphics usually contains both dynamic and static granular phenomena. Discrete element methods (DEM) [BYM05, ATO09] are widely used in animating such scenes. However, the computational efficiency and scalability of DEM is still a challenge. In each time step, interparticle interactions need to be computed for all the particles in the scene, both visible and invisible; mobile and immobile. Because sand is opaque, the appearance of a sand pile is governed by its surface layer while its interior is invisible. And in the process of heap formulation, flow only exists in a surface layer while the interior bulk is immobile. So if the large unseen and immobile region of the sand piles can be represented as efficiently as possible (e.g. using a height field) without losing physical accuracy, and the main computation resources are allocated to the visually important pile surface and the highly dynamic grains in space, the computational efficiency will improve significantly.

Based on this idea, we propose a new sand animation method: animating sand as a surface flow. The general philosophy of our method is to subdivide the accumulated sand in the scene into a surface flowing layer and a static layer in a physically accurate way. Three dimensional discrete element method is used in the surface layer where we expect interesting flowing details, while the invisible static layer is represented by a 2D height field for efficiency. To separate the two layers accurately, the BCRE model (Bouchaud, Cates, Ravi Prakash, and Edwards [BCRE94]) is used in our approach. The BCRE model is a typical surface flow model of granular material used in civil engineering to describe the "rolling" and "static" phase inside the granular packing. In our method, this height field model is incorporated into an existing DEM solver with only a few additions, and guides the evolution of the entire sand system by the matter transfer process between the two layers.

2. Related Work

Bell et al. [BYM05] were the first to introduce DEM into sand animation in computer graphics. Many interesting 3D effects were provided in their work, yet with very high computational cost. Since then, several efforts have been made to accelerate DEM, including multi-resolution method [ATO09], advecting particles on the eroded surface [ABC*07], ignoring calculations of contact forces among unmoved particles [MAS*09] and etc. Additionally, Yongning Zhu et al. [ZB05] and Lenaerts et al. [LD09] simulate sand as a continuum from the viewpoints of Euler and Lagrange respectively. Their methods avoid the computation of each physical grain and are appropriate scalable.

Height field methods [ON03] animate sand as a 2D terrain and can produce interesting effects such as footprints and tire tracks. But their abilities in animating 3D effects are limited. The BCRE model has also been introduced into the computer graphics field as a height field model. Pla-Castells et al. [PC08] uses cellular automata to model interactive terrain systems based on the BCRE equations. But their method is only suitable for terrain simulation and cannot model the 3D dynamic behaviors of sand. In fluid animation field, there also exist methods coupling 2D and 3D techniques to improve the simulation efficiency. In [IGLF06], Irving et al. used height field to represent large bulk of water and 3D Navier-Stokes solver to simulate the detailed water surface.

3. Method

In our method, the motion of the granular media is represented by discrete particles as in standard DEM. As illustrated in Figure 1, the mobile particles, including particles in the flowing layer and particles scattered in space, are maintained in an active particle list. The immobile particles in the static layer are reduced and represented by the 2D height field columns. For each time step Δt, our sand simulation method contains three main steps. First, the two-layer height field model is evolved based on the BCRE equations. Second, the matter transfer between mobile particles and height field columns is modeled through a particle interface, and the elements in the active list are refreshed. Third, the positions and velocities of the mobile particles in the active list are updated with a DEM solver to describe the motion behaviors in the scene.

3.1. Evolution of the flowing and static layer with the BCRE Model

In the first step, the BCRE surface flow model [BCRE94, ARdG02] is used to separate granular piles into a rolling phase composed of mobile grains atop a static phase composed of height field columns, with a sharp interface. The variables used in the BCRE model are illustrated in Figure 1. The variation of the thickness of the flowing layer $R(x,t)$ and the thickness of the static phase $h(x,t)$ are expressed by the following equations:

$$\frac{\partial h}{\partial t} = -\varepsilon(x,t) \quad (1)$$

$$\frac{\partial R}{\partial t} = v_d \frac{\partial R}{\partial x} + \varepsilon(x,t). \quad (2)$$

In which, v_d is the local velocity of the rolling grains, and $\varepsilon(x,t)$ is the exchange term, which represents the matter transfer that occurs between the two phases. In our method, we use the following expression mentioned in [ARdG02]:

$$\varepsilon(x,t) = \gamma R(\theta - \theta_0). \quad (3)$$

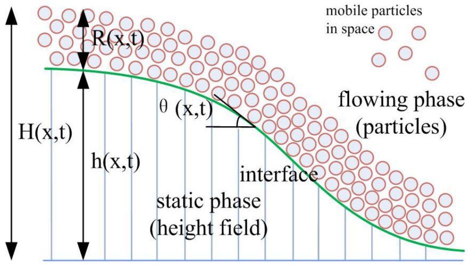


Figure 1: A sand pile is subdivided into two layers based on the BCRE model. The surface flowing layer is represented by particles and the static layer is represented by height field.

In equation 3, γ denotes the exchange rate between the two phases, θ is the slope angle of the interface and θ_0 is the repose angle of the granular material. The exchange ensures that the interface slope angle not exceeds the granular repose angle.

We use a uniform 2D horizontal grid of vertical columns to store the BCRE height field information. Similar to the MAC grid used in Euler fluid [IGLF06], H, h, R and Δh are stored at cell centers and the slope angles θ are stored on the respective edges. For each time step Δt, the sequence of computation of the four variables is:

$$H \rightarrow h \rightarrow \Delta h \rightarrow h \rightarrow R \rightarrow R' \rightarrow h'. \quad (4)$$

First, as in [ABC*07], H is calculated as the height of the sand continuum in 3D space via ray casting. This ensures that the total volume of the sand pile is only related to the accumulated particles in the scene, rather than the evolving h and R. Then, the slope angle θ on the cell edge is calculated as $\arctan(h_{nb} - h)/d$, and the derivative of h is calculated using equation 3. In the next step, h and R are temporally evolved with the exchange term. At last, the thickness of surface layer needs to be no less than a threshold and the values of R and h are adjusted according to this constraint.

To model the granular media flowing on irregular terrains or accumulating in containers (see Figure 4a, 4b), a multi-valued height field L similar to [ON03] is predefined to represent the complex environment, and the height field variables are calculated with the terrain heights. However, the terrain height field can only help to model the evolution of h and R in the BCRE equations. To model the interactions between grain particles and environment accurately (see Figure 4c, 4d), the particle surface proposed in [BYM05] is used to represent the rigid objects and terrains with complex shapes.

3.2. Matter transfer between the two layers

The matter transfer process between the two phases is a key part of the surface flow model. The static/mobile frontier is not predefined as an optical depth for the coercively coupling of the two layers, instead, it is a dynamic variable evolved with the BCRE equations. In our approach, the matter transfer is modeled by particle erosion and deposition. As in equation 3, when $\theta > \theta_0$, there is a net erosion from static layer to flowing layer and the static layer height h will locally decrease. When $\theta < \theta_0$, there is a net deposition from flowing layer to static layer and h will locally increase.

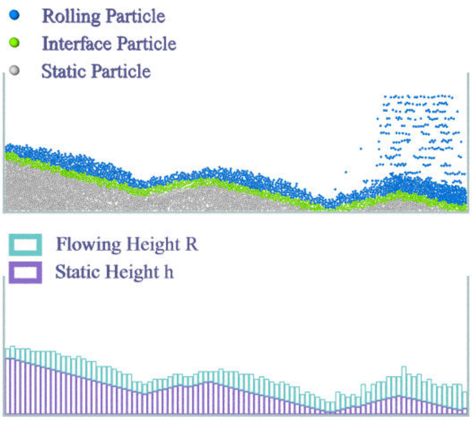


Figure 2: The two layers of the sand system are distinguished based on the BCRE height field. A particle interface (green particles) with constant thickness is maintained to separate the two layers.

To couple the different representations of particle and height field, we model the erosion and deposition process by maintaining an evolving particle interface with constant thickness between the two layers. As illustrated in Figure 2, the interface is controlled by the BCRE height field, and consists of particles with fixed positions. The particle interface acts as a buffer region between the rolling particles and the static height field columns to keep the exchange process stable. When h rises and the material of rolling layer is deposited onto the bottom, the initial mobile particle near the interface are trapped and marked as interface particles. When h descends and the bottom layer is eroded by the surface flow, the fixed particles near the flowing layer are dislodged and brought into motion again. Once the particles are trapped into the interface, their positions are fixed, yet they can still affect the motion of the rolling particles by imposing normal and tangent molecule forces on the neighboring mobile particles.

The particles in the static layer beneath the interface are deleted from the active particle list. Their volumes are represented by the height field columns and their positions are stored in the spatial cells in hashing grid. In the last step, only the mobile particles in the active list (including particles in the surface layer and particles moving in space) need to be updated by a DEM solver. Since a large number of particles with high contact forces in the bottom part are reduced, the stiffness of the entire system in time integration is decreased, which makes larger time steps possible for the simulation. This speed-up is more obvious when the heaps are becoming larger and higher.

4. RESULTS AND DISCUSSION

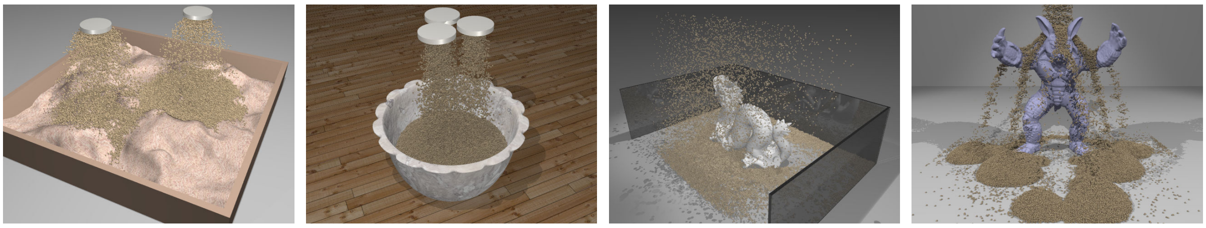
We have implemented our method to create several test scenes. The physics simulation framework was implemented in C++ and the animations were rendered using Pov-Ray. All simulations ran on a dual-core 2.66GHz CPU and 4GB of memory.

Figure 3: Sand simulation using standard DEM (left) and our approach (right).

Figure 3 shows sand grains falling onto a flat floor and formulating heaps. We use our approach and the standard DEM to simulate the same scene. The results show that the two methods achieve very similar simulation effects. But our approach only use 28k surface particles for simulation, compared to 107k in conventional DEM. Our approach takes approximately 9s per frame on average, compared to 68s in standard DEM method.

In Figure 4a, sand grains are splashed and accumulate in a box with terrain. This example shows that our approach works on an irregular terrain. Figure 4b shows granular grains being poured into a bowl. Since a large bulk of sand in the bowl can be represented by a height field, only 25k surface particles are used to simulate the accumulating process of the 318k particles contained in the bowl. Figure 4c and Figure 4d show two complex scenes including both highly dynamic and static granular phenomena. In Figure 4c, sand waves are generated with high initial horizontal velocity and are halted by the dragon model and the glass wall. Heaps are produced on the ground near the dragon and near the base of the wall. In Figure 4d, sand grains are poured onto the armadillo and formulate heaps with different sizes on the ground.

As shown in Table 1, the particle number used in simulation has been reduced to 8%-30% by using the surface flow model. Since a large bulk of grains with high contact forces are represented by height field, our approach does not suffer from the rather small time steps as in standard DEM. The average time steps are ranging from 0.3-1.0 ms in our simulations. Because a large part of particles are reduced, and the computational cost of the height field model is trivial,



(a) (b) (c) (d)

Figure 4: (a)Sand grains are splashed onto an irregular terrain. (b)Sand grains are poured into a bowl. (c)Sand grains are splashed and halted by the dragon and the glass wall. (d)Sand grains are poured onto the armadillo.

Table 1: Particle numbers and timing data

Scenes	Total	Surface	Time(s/frame)
Floor	107,687	28,410	8.62
Terrain	199,039	48,423	18.27
Bowl	318,053	24,710	11.01
Dragon	192,975	40,043	30.05
Armadillo	207,020	58,540	21.54

our method provides a significant speed-up compared to the results in [BYM05] and [ATO09].

There are also some limitations of our approach. Since our method only simplifies the unseen and static part of sand system, the efficiency gains depend on the characteristics of the sand flow. When most of the sand grains in the scene are highly dynamic, or when the distribution of grains in the space is very sparse and few heaps are formulated, smaller performance improvement is gained and most of the particles need to be updated using the DEM solver.

5. Conclusion

In this paper we present a novel efficient approach to simulate sand and other granular materials in 3D scenes. Our method couples the 2D and 3D simulation techniques by using the BCRE surface flow model. The visible and rolling part of the sand flow is simulated using discrete element method and the invisible and static part is represented by height field. The physics accuracy of the model is maintained by the matter transfer process, which is the key part of the BCRE equations, through a particle interface between the two layers. Our method performs well in large-scale 3D scenes where the motion states of sand grains transfer quickly, and leads to substantial savings of both memory and computation time.

6. Acknowledgement

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