$$
\begin{aligned}
\mathrm{E}[L] &= \sum_{j=0}^{n} j \Pr\{L_j\} \\
&= \sum_{j=0}^{\lfloor(\lg n)/2\rfloor-1} j \Pr\{L_j\} + \sum_{j=\lfloor(\lg n)/2\rfloor}^{n} j \Pr\{L_j\} \\
&\geq \sum_{j=0}^{\lfloor(\lg n)/2\rfloor-1} 0 \cdot \Pr\{L_j\} + \sum_{j=\lfloor(\lg n)/2\rfloor}^{n} \lfloor(\lg n)/2\rfloor \Pr\{L_j\} \\
&= 0 \cdot \sum_{j=0}^{\lfloor(\lg n)/2\rfloor-1} \Pr\{L_j\} + \lfloor(\lg n)/2\rfloor \sum_{j=\lfloor(\lg n)/2\rfloor}^{n} \Pr\{L_j\} \\
&\geq 0 + \lfloor(\lg n)/2\rfloor (1 - O(1/n)) \qquad \text{(by inequality (5.11))} \\
&= \Omega(\lg n) .
\end{aligned}
$$

As with the birthday paradox, we can obtain a simpler but approximate analysis using indicator random variables. We let $X_{ik} = \mathrm{I}\{A_{ik}\}$ be the indicator random variable associated with a streak of heads of length at least k beginning with the ith coin flip. To count the total number of such streaks, we define

$$
X = \sum_{i=1}^{n-k+1} X_{ik} .
$$

Taking expectations and using linearity of expectation, we have

$$
\begin{aligned}
\mathrm{E}[X] &= \mathrm{E}\left[\sum_{i=1}^{n-k+1} X_{ik}\right] \\
&= \sum_{i=1}^{n-k+1} \mathrm{E}[X_{ik}] \\
&= \sum_{i=1}^{n-k+1} \Pr\{A_{ik}\} \\
&= \sum_{i=1}^{n-k+1} 1/2^k \\
&= \frac{n-k+1}{2^k} .
\end{aligned}
$$