Approximation Algorithms

EO249

(Problem Set 3) Due: Feb 27th, 2015

Solutions need to be submitted by email to eo249iisc@gmail.com. We prefer latexed solution. Before giving the solutions, you should write how many problems you have attempted and how many you think you have solved. Starred problems are optional and (possibly) more fun.

Exercise 1. In the KNAPSACK problem, we are given a bound B which is an integer, a set of n items with each item j having an integer weight w_j and an integer profit p_j. The goal is to choose a subset S of items such that $\sum_{j \in S} w_j \leq B$ and $\sum_{j \in S} p_j$ is maximized.

1. Consider the greedy algorithm: order the items in decreasing order of p_j/w_j and pick items in this order till you fill the knapsack. What is the approximation factor of this algorithm?
2. Design a polynomial time algorithm for this problem if $p_j \leq n$ for all j.
   (Hint: Dynamic programming.)
3. Design a polynomial time approximation scheme (PTAS) for the knapsack problem.
   (Hint: Scale each p_j by a factor such that the new p_j's are all less than n/ε. Then argue that the optimum value of the scaled instance can't be too much smaller than that of the original instance.)

Exercise 2. Describe an **exact** algorithm for TSP on n-points running in time $O(2^n \text{poly}(n))$.

Exercise 3. Describe a PTAS for Euclidean TSP in d-dimensions (we did $d = 2$ in class). What is the dependence on d?