

E0234: Assignment 6

Due: Monday, 22nd Feb 2016.

It is highly recommended you do not google for the answers to the questions below. You can discuss with your friends, but then mention that in your submission. The writing should solely be your own.

1. In the permutation routing problem on the hypercube, consider the following randomized oblivious protocol. Each x considers a random order of $\{1, 2, \dots, n\}$ and then bit-fixes to get to $\pi(x)$ in that order. Show that this protocol has expected makespan of $2^{\Omega(n)}$.
2. For the packet routing and scheduling problem on fixed paths, consider the “active” protocol which tries to move the packets as soon as possible, that is, packets start routing immediately waiting at vertices in a fixed waiting order in case of collisions. We argued in class this has makespan $O(CD)$. What is the best *lower bound* you can prove on the makespan?
3. In class, we saw that a random cut in an undirected graph has expected cut size $|EG|/2$. What is the best lower bound you can prove on the probability that the random cut is $\geq |EG|/2$?
4. A legal coloring of a graph G with c colors assigns a color in $\{1, 2, \dots, c\}$ to every vertex such that for all edges (u, v) , the color of u and color of v are different. The *chromatic number* $\chi(G)$ of a graph G is the minimum number of colors with which a graph can be legally colored. Prove that for any integers g and c , there exists a graph G with girth $\geq g$ and $\chi(G) \geq c$.