CS 49/149: Approximation Algorithms

Problem set 0. Due: Not for submission.

General small print: Please submit all homework electronically in PDF format ideally typeset using LaTeX. You need to submit only the problems above the line. Please try to be concise – as a rule of thumb do not take more than 1 (LaTeX-ed) page for a solution. We highly encourage students to also do the problems below the line for a better understanding of the course material.

Topics in this HW: This particular assignment is not for submission. You should be able to do all the problems (with possibly some work – the trickier ones are starred) if you have done an undergraduate algorithms course.

Problem 1. Given any tree, prove that there are an even number of vertices whose degree is an odd number. Do the number of vertices with even degree need to be even as well?

Problem 2. We are given a directed graph $G = (V, E)$ with non-negative costs $c(e)$ on every edge. There is a source vertex s and a destination vertex t. Give an efficient algorithm for finding the path from s to t with the smallest total cost, which visits at most 10 other vertices on the way.

Problem 3. Which of the following are NP hard, and which have polynomial time algorithms? Answer with either a reduction or an algorithm.

- Given a graph G, decide whether or not it contains a cycle of *odd* length.
- Given a set of n integers in the range $[-n, +n]$ and a target B, decide whether or not there is a subset of these which add up to exactly B.
- Given a set of m linear equations in n variables and integer coefficients in the range $[-n, +n]$, decide whether or not
  a) there is an assignment of the variables to reals which satisfy all the equations.
  b) (*) there is an assignment of the variables to $\{0, 1\}$ which satisfy all the equations.

Problem 4. (*) Given a directed graph $G = (V, E)$ with costs $c(e)$ on edges, a *cycle cover* is a subset $F \subseteq E$ of the edges such that (a) each component of (V, F) is a cycle, and (b) each vertex of V is in exactly one cycle. The cost of the cycle cover is the sum of the costs of the edges in F. Design a polynomial time algorithm to find the minimum cost cycle cover of a graph G.

Hint: Consider the bipartite graph H with bipartition $A \cup B$ where A and B are copies of vertices of G, and there is an edge from $u \in A$ to $v \in B$ if and only if (iff) there is a directed edge from u to v in G. What do cycle covers in G correspond to in H?