CS 49/149: Approximation Algorithms

Problem set 3. Due: 22nd April, 6:59pm

General small print: Please submit all homework electronically in PDF format ideally typeset using LaTeX. You need to submit only the problems above the line. Please try to be concise – as a rule of thumb do not take more than 1 (LaTeX-ed) page for a solution. We highly encourage students to also do the problems below the line for a better understanding of the course material.

Topics in this HW: LP Relaxations

Problem 1. Prove that for *bipartite* graphs, the natural LP relaxation for vertex cover is exact. That is, there is always a vertex cover in the graph equal to the LP-value.

Problem 2. Consider the vertex cover problem when all costs $c_v = 1$ and the graph is d-regular (all vertices have degree d). Prove that the integrality gap of the normal LP-relaxation is $\leq 2(1 - \frac{1}{d+1})$.

Problem 3. Prove that the *integrality gap* of the Set-Cover LP done in class in $\Omega(\log n)$ where n is the number of elements. Hint: Think of the elements as bit-vectors in d-dimensions.

Problem 4. Design a valid LP-relaxation for the *shortest path* problem in a directed graph. Prove your relaxation is exact when the costs on arcs are non-negative.

Problem 5. Modify the algorithm for Facility Location done in class to obtain a 4-approximation algorithm.

Problem 6. Describe a greedy $O(\log n)$-approximation algorithm for the facility location problem when the costs need not form a metric.

Problem 7. Design an LP-relaxation for the Traveling Salesman Problem. Prove upper bounds and lower bounds on its integrality gap.

Problem 8. (*) Construct an example of a factor 2-integrality gap for the stronger relaxation to the vertex cover problem where we add the constraints

$$\forall u, v, w : (u, v), (v, w), (w, u) \in E, \quad x_u + x_v + x_w \geq 2$$