# What should I know before taking CS 30

- **Boolean Variables.** Boolean variables or simply Booleans are the most basic unit of data: the *bit*. The bit takes two values: $1$ or "True", and $0$ or "False".

- **Numbers and Arithmetic.** There are many types of numbers: *natural/whole numbers* $\{0, 1, 2, 3, \ldots, \}$, *integers* $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$, which also include negative numbers, *rational numbers*, which are of the form $p/q$ where both $p$ and $q$ are integers, and *real* numbers which can represented on the number line can be approximately represented by *decimals*.

- **Arithmetic.** Any two numbers can be added, subtracted, multiplied, and divided. Rationals are *closed* under these *operations*, that is, the sum/difference/product/ratio of any two rationals is rational. This is *not true* for integers. For instance, $2$ divided by $4$ is not an integer.

- **Associativity and Comuutativity.** Addition and Multiplication are *associative* and *commutative*. That is, $(a + b) + c$ is the same as $a + (b + c)$ and this is succinctly written as $a + b + c$. Similarly, $(a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot b \cdot c$.

  This is *not true* for subtraction. $(a - b) - c$ is not the same as $a - (b - c)$. Indeed, the latter is $(a - b) + c$.

- **Multiplication distributes over addition and subtraction.** For any three numbers, $a \cdot (b + c)$ equals $a \cdot b + a \cdot c$. Similarly, $a \cdot (b - c) = a \cdot b - a \cdot c$.

- **Prime and Composite Numbers.** A positive number $p$ is *prime* if the only numbers dividing it (that is, leaving $0$ remainder) are $1$ and $p$. Otherwise, the number is composite. The only exception to the rule is $1$ which is neither prime nor composite. For instance, 17 is a prime, but $323 = 17 \times 19$ is not.

- **Prime Factorization.** Any composite number $n$ can be *uniquely* written as a product of primes (unique upto changing the ordering of multiplication). This is the *prime factorization theorem*. This is a non-trivial fact which we may or may not prove in CS 30.

- **Floors and Ceilings.** Given any real number $x$, the *floor* $\lfloor x \rfloor$ is the *largest integer smaller than or equal to* $x$. So, $\lfloor 2 \rfloor = 2$, and $\lfloor 1.5 \rfloor = 1$, and $\lfloor -2.7 \rfloor = -3$. Similarly, the *ceiling* $\lceil x \rceil$ is the *smallest integer larger than or equal to* $x$. So, $\lceil 2 \rceil = 2$ and $\lceil 1.5 \rceil = 2$ and $\lceil -2.7 \rceil = -2$.

- **Exponentiating.** Given any positive integer $n$ and any real $x$, the number $x^n$ is a shorthand for $x \cdot x \cdot x \cdots x$, where $x$ is multiplied with itself $n$ times. So, $3^2 = 9$ and $(1.5)^2 = 2.25$ and $(-1)^3 = -1$.

  The number $x^0$ is defined to be $1$ for *any* $x$. For any negative number $y = -z$, we define $x^y = x^{-z} := 1/x^z$.

  Some properties of exponentials: For any reals $x, y$ and numbers $a, b$, we have

  1. $x^{a+b} = x^a \cdot x^b$
  2. $(xy)^a = x^a \cdot y^a$
  3. $(x^a)^b = x^{ab}$

- **Logarithms.** The *logarithm* of a *positive* number $a$ to the *positive base* $b \neq 1$, denoted as $\log_b a$ is the number $\ell$ such that $b^\ell = a$. Thus, $\log_3 81 = 4$ (since $3^4 = 81$) and $\log_{4/3}(16/9) = 2$ (since $(4/3)^2 = 16/9$), and $\log_{\sqrt{2}} 16 = 4$ (since $(\sqrt{2})^8 = 16$).

  Some properties of logarithms: These properties follow by using the properties of exponential and the definition of logarithms

  1. $\log_b 1 = 0$ for all $b > 0$.
  2. $\log_b(xy) = \log_b x + \log_b y$ for all $b, x, y > 0$.
  3. $\log_b(x^y) = y \log_b x$ for all $b, x, y > 0$.
  4. $\log_b x = \frac{\log_c x}{\log_c b}$.

- **Basic Formulae.** You should be able to deduce the following

  1. For any two numbers, $(a+b)^2 = a^2 + 2ab + b^2$
  2. For any two numbers, $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
  3. For any three numbers, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

- **Polynomials.** A polynomial is a formula of the form $p(x) = a_d x^d + a_{d-1}x^{d-1} + \cdots + a_1 x + a_0$. $x$ is called the *variable*. The *degree* of a polynomial is the largest power of $x$ which participates in the polynomial. In the above example, this is $d$. The constants $a_0, a_1, \ldots, a_d$ are the *coefficients* of the polynomial.

  For instance, $p(x) = x^2 + 2x + 1$ is a polynomial, and so is $q(x) = 5x^3 - 7x + 4$. The degree of $p(x)$ is 2 and its coefficients are $1, 2, 1$, while the degree of $q(x)$ is 3 and its coefficients are $5, 0, -7, 4$.

You should be able to solve these problems. If not, come talk to me ASAP.

- What is the value of (a) $\lfloor 2.5 \rfloor + \lceil 3.75 \rceil$? (b) $(\lfloor \pi \rfloor)^{\lceil \pi \rceil}$?
- Are $1 + \lfloor x \rfloor$ and $\lfloor 1 + x \rfloor$ always equal? Are $\lceil \lfloor x \rfloor \rceil$ and $\lfloor \lfloor x \rfloor \rfloor$ always equal?
- If $x$ and $y$ are rational numbers, are $x + y$, $x - y$, and $x \cdot y$ always rational? How about $x/y$?
- Which is bigger $3^{10}$ or $10^3$?
- What is the value of (a) $\log_{1/8} 2$? (b) $\log_2 16$?
- Which is bigger, $\log_{10} 17$ or $\log_{17} 10$?