What should I know before taking CS 30

- **Boolean Variables.** Boolean variables or simply Booleans are the most basic unit of data: the bit. The bit takes two values: 1 or “True”, and 0 or “False”.

- **Numbers and Arithmetic.** There are many types of numbers: natural/whole numbers \{0, 1, 2, 3, \ldots\}, integers \{\ldots, -2, -1, 0, 1, 2, \ldots\}, which also include negative numbers, rational numbers, which are of the form \( p/q \) where both \( p \) and \( q \) are integers, and real numbers which can represented on the number line can be approximately represented by decimals.

- **Arithmetic.** Any two numbers can be added, subtracted, multiplied, and divided. Rationals are closed under these operations, that is, the sum/difference/product/ratio of any two rationals is rational. This is not true for integers. For instance, 2 divided by 4 is not an integer.

- **Associativity and Comutativity.** Addition and Multiplication are associative and commutative. That is, \((a + b) + c\) is the same as \(a + (b + c)\) and this is succinctly written as \(a + b + c\). Similarly, \((a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot b \cdot c\).

This is not true for subtraction. \((a - b) - c\) is not the same as \(a - (b - c)\). Indeed, the latter is \((a - b) + c\).

- **Multiplication distributes over addition and subtraction.** For any three numbers, \(a \cdot (b + c)\) equals \(a \cdot b + a \cdot c\). Similarly, \(a \cdot (b - c) = a \cdot b - a \cdot c\).

- **Prime and Composite Numbers.** A positive number \(p\) is prime if the only numbers dividing it (that is, leaving 0 remainder) are 1 and \(p\). Otherwise, the number is composite. The only exception to the rule is 1 which is neither prime nor composite. For instance, 17 is a prime, but 323 = 17 \times 19 is not.

- **Prime Factorization.** Any composite number \(n\) can be uniquely written as a product of primes (unique upto changing the ordering of multiplication). This is the prime factorization theorem. This is a non-trivial fact which we may or may not prove in CS 30.

- **Floors and Ceilings.** Given any real number \(x\), the floor \([x]\) is the largest integer smaller than or equal to \(x\). So, \([2]\) = 2, and \([1.5]\) = 1, and \([-2.7]\) = -3. Similarly, the ceiling \([x]\) is the smallest integer larger than or equal to \(x\). So, \([2]\) = 2 and \([1.5]\) = 2 and \([-2.7]\) = -2.

- **Exponentiating.** Given any positive integer \(n\) and any real \(x\), the number \(x^n\) is a shorthand for \(x \cdot x \cdot x \cdots x\), where \(x\) is multiplied with itself \(n\) times. So, \(3^2 = 9\) and \((1.5)^2 = 2.25\) and \((-1)^3 = -1\).

The number \(x^0\) is defined to be 1 for any \(x\). For any negative number \(y = -z\), we define \(x^y = x^{-z} := 1/x^z\).

Some properties of exponentials: For any reals \(x, y\) and numbers \(a, b\), we have

1. \(x^{a+b} = x^a \cdot x^b\)
2. \((xy)^a = x^a \cdot y^a\)
3. \((x^a)^b = x^{ab}\)
• **Logarithms.** The logarithm of a positive number \( a \) to the positive base \( b \neq 1 \), denoted as \( \log_b a \) is the number \( \ell \) such that \( b^\ell = a \). Thus, \( \log_3 81 = 4 \) (since \( 3^4 = 81 \)) and \( \log_{4/3} (16/9) = 2 \) (since \( (4/3)^2 = 16/9 \)), and \( \log_{\sqrt{2}} 16 = 4 \) (since \( (\sqrt{2})^8 = 16 \)).

Some properties of logarithms: These properties follow by using the properties of exponential and the definition of logarithms

1. \( \log_b 1 = 0 \) for all \( b > 0 \).
2. \( \log_b (xy) = \log_b x + \log_b y \) for all \( b, x, y > 0 \).
3. \( \log_b (x^y) = y \log_b x \) for all \( b, x, y > 0 \).
4. \( \log_b x = \frac{\log_a x}{\log_a b} \).

• **Basic Formulae.** You should be able to deduce the following

1. For any two numbers, \((a + b)^2 = a^2 + 2ab + b^2\)
2. For any two numbers, \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\)
3. For any three numbers, \((a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)\)

• **Polynomials.** A polynomial is a formula of the form \( p(x) = a_d x^d + a_{d-1} x^{d-1} + \cdots + a_1 x + a_0 \). \( x \) is called the variable. The degree of a polynomial is the largest power of \( x \) which participates in the polynomial. In the above example, this is \( d \). The constants \( a_0, a_1, \ldots, a_d \) are the coefficients of the polynomial.

For instance, \( p(x) = x^2 + 2x + 1 \) is a polynomial, and so is \( q(x) = 5x^3 - 7x + 4 \). The degree of \( p(x) \) is 2 and its coefficients are 1, 2, 1, while the degree of \( q(x) \) is 3 and its coefficients are 5, 0, -7, 4.

You should be able to solve these problems. If not, come talk to me ASAP.

• What is the value of (a) \([ 2.5 ] + [ 3.75 ]\)? (b) \( ( [ \pi ] ) ^ { [ \pi ] } \)?
• Are \( 1 + [ x ] \) and \( [ 1 + x ] \) always equal? Are \( [ [ x ] ] \) and \( [ [ x ] ] \) always equal?
• If \( x \) and \( y \) are rational numbers, are \( x + y \), \( x - y \), and \( x \cdot y \) always rational? How about \( x/y \)?
• Which is bigger \( 3^{10} \) or \( 10^{3} \)?
• What is the value of (a) \( \log_{1/8} 2 \)? (b) \( \log_{2} 16 \)?
• Which is bigger, \( \log_{10} 17 \) or \( \log_{17} 10 \)?