Bing Chat Flunks Math

Baby math

Me: What is the time difference between Ontario, OR, and Pensacola, FL? What is the minimum time difference between OR and FL?

Bing Chat: The time difference between Ontario, OR and Pensacola, FL is 1 hour [1]. Pensacola is 1 hour ahead of Ontario [1].

Florida observes only one time zone, which is the Eastern Time Zone [2]. Oregon, on the other hand, observes two time zones: Pacific Time Zone and Mountain Time Zone [2].

The minimum time difference between Oregon and Florida is 3 hours [3]. Salem, Oregon is 3 hours behind Tallahassee, Florida [3].


Even with the big hint of the first question, Chat stubbornly gets the second one wrong. It’s really dumb to say the minimum time difference is greater than a stated actual time difference. Reference [2] may tell you that Oregon has two time zones, but it will not tell you that Florida has only one.

Reference [3] will not make the first assertion attributed to it. Clocks in the capital cities, Salem and Tallahassee, are not set to the same times as clocks in Ontario and Pensacola. If you ask [3] what time it is in Florida, it gives two times: that in Pensacola and that in Tallahassee.

Freshman calculus

I fed Bing Chat the same elementary calculus problem eight times and got eight different answers, none fully right. It never hinted that its math is suspect because it can’t reason, although its “derivations” have the trappings of mathematical argument.

Chat can talk like a mathematician but it can’t think like one. Press accounts often say that LLMs can “hallucinate”. It is far more damning to observe that Chat can’t reason. Unfortunately, it never expresses uncertainty.

My question to Chat was, “What are the coordinates of points where the magnitude of the slope of the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is one?”

A derivation of the correct answer appears in the Appendix.

Chat obviously was familiar with the equation of the ellipse and the convention that \( x \) and \( y \) are the variables whose values are sought. I was impressed that it coped with renaming \( a \) and \( b \), but the results below show that it is bad at substitution in general. The big take-away is that the math is typically done by guess and is not carried through as described in the accompanying text.
Chat’s representation of math formulas differed from trial to trial. Being interested in math, and not with how Chat interfaces with HTML, I have quietly reformatted formulas for readability. I have also rearranged Chat’s layouts for consistency. Chat’s text, however, has been preserved.

Chat’s text is shown in a fixed-width font. Math errors—formulas that do not follow from previous (possibly incorrect) formulas are flagged ! !.

**Trial 1, Fri, 13 Oct 2023**

*Chat almost always stated the slope of the ellipse right, usually by simple assertion, as here. Probably the formula came straight from the training set. This trial runs wild from there on.*

The equation of the ellipse is given by

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \]

The slope of the ellipse at any point \((x, y)\) is given by

\[ -\frac{b^2 x}{a^2 y}. \]

The magnitude of the slope is

!! \( \left(\left(\frac{b^4 x^2}{a^4 y^2}\right)/\left(a^4 y^2\right) + 1\right)^{1/2}. \)

*How did Chat come up with this magnitude—the length of the hypotenuse of an axis-oriented right triangle in which the hypotenuse lies on the tangent line and meets a leg of length 1 on the x axis?*

We need to find the points where the magnitude of the slope is 1.

*Chat next proposes to differentiate the slope with respect to \(y\)! How is this second derivative made into an equation?*

Differentiating the magnitude of the slope with respect to \(y\) and simplifying, we get

!! \( b^4 x^2 = a^4 y^2 \left(1 - \left(b^4 x^2\right)/\left(a^4 y^2\right)\right). \)

*Tellingly, no fractional powers or odd powers appear in the supposed derivative of the square root of a rational function that contains only even powers.*

Simplifying further, we get

!! \( a^4 y^4 - b^4 x^2 y^2 + b^4 x^2 a^4 = 0. \)

*The prior equation can be cast as a homogeneous equation of total degree 12. Chat’s “simplification” magically turns it into an inhomogeneous equation.*

This is a quadratic equation in \(y^2\) with coefficients \(a^4\), \(-b^4 x^2\), and \(b^4 x^2 a^4\). Solving this quadratic equation, we get

!! \( \pm\left(\frac{b}{a}\right)^4 - x^2)^{1/2}. \)

*After Chat’s astute observation, one expects the quadratic formula to be used. However, the rational part of the quadratic formula is missing and the square-root part is wrong.*

Therefore, the coordinates of the points where the magnitude of the slope is 1 are
\( (x, \pm ((b/a)^4 - x^2)^{1/2}). \)

The result defies dimensional analysis; \(b/a\) is a pure number while \(x\) is a length. And \(x\) is left unspecified; we are given a locus, not a finite set of points.

Please note that this is a general solution for all ellipses. For a specific ellipse, you need to substitute the values of \(a\) and \(b\) in the above expression.

**Trial 2, Mon, 16 Oct 2023**

The answer here also defies dimensional analysis; lengths \(x\) and \(y\) are equated to areas \(a^2\) and \(b^2\).

The equation of the ellipse is
\[
x^2/a^2 + y^2/b^2 = 1.
\]

The slope of the tangent line to the ellipse at a point \((x, y)\) is given by
\[
-\frac{x*b^2}{(y*a^2)}.
\]

The magnitude of the slope of the tangent line is 1 when \(x = a^2\) and \(y = b^2\).

This point satisfies the slope condition, but no reason is given to believe the point is on the tangent line or on the ellipse. In full generality, the slope condition is satisfied when the magnitude of \(x\) is to the magnitude of \(y\) as \(a^2\) is to \(b^2\), i.e. when \((x, y) = (\pm ka^2, \pm kb^2)\) for positive \(k\). The Appendix shows that the value of \(k\) when \((x, y)\) is on the ellipse is \((a^2 + b^2)^{-1/2}\).

Therefore, the coordinates of the points where the magnitude of the slope of the ellipse is one are:
\[
(x, y) = (a^2, b^2), (-a^2, -b^2), (-a^2, b^2), (a^2, -b^2).
\]

I will give Chat the benefit of the doubt for not explaining the last three points.

**Trial 3, Tue, 17 Oct 2023**

This answer is unique for citing a reference. However, Chat improperly attributes its mistakes as well as its borrowings.

The equation of the ellipse is [1]
\[
x^2/a^2 + y^2/b^2 = 1
\]

The slope of the ellipse at any point \((x, y)\) is given by [1]
\[
\frac{dy}{dx} = -\frac{b^2*x}{(a^2*y)}
\]

The magnitude of the slope is given by [1]
\[
|\frac{dy}{dx}| = b^2*x/a^2*y
\]

Taking the magnitude is incorrectly conflated with sign reversal. The error will subsequently be hidden by squaring. But not all the final answers will satisfy \(b^2*x/a^2*y = 1\).

We need to find the points on the ellipse where \(|dy/dx| = 1\).
The next sentence might charitably be understood to mean solve \( |dy/dx| = 1 \) for \( y \) in terms of \( x \) and substitute that into the equation of the ellipse. However, the results apparently come not from substitution but from arbitrarily setting to 1/2 each term on the left hand side of the equation of the ellipse.

Substituting \( |dy/dx| = 1 \) in the equation of the ellipse, we get \([1]\)

\[
| dy / dx | = 1 \implies x = a \sqrt{2} \text{ or } x = -a \sqrt{2}
\]

Substituting these values of \( x \) in the equation of the ellipse, we get \([1]\)

\[
y = b \sqrt{2} \text{ or } y = -b \sqrt{2}
\]

Therefore, the coordinates of the points where the magnitude of the slope of the ellipse is one are \([1]\)

\[
(a \sqrt{2}, b \sqrt{2}), \ (-a \sqrt{2}, -b \sqrt{2}), \ (-a \sqrt{2}, b \sqrt{2}), \text{ and } (a \sqrt{2}, -b \sqrt{2})
\]


Trial 4, Wed, 18 Oct 2023

The only formula below that makes sense is the squaring step. Even the radius of the final circle is incorrectly calculated from its square, defying dimensional analysis, as did Trials 1 and 2.

We want to find the points on the ellipse where the magnitude of the slope is 1. Therefore, we have

\[
| dy / dx | = 1 \implies \frac{b \sqrt{a^2 + b^2}}{a \sqrt{x^2 + y^2}} = 1
\]

Squaring both sides, we get:

\[
\frac{b^2(a^2 + b^2)}{a^2(x^2 + y^2)} = 1
\]

Rearranging, we get:

\[
x^2 + y^2 = \frac{a^2 b^2}{\sqrt{a^4 - b^4}}
\]

This is the equation of a circle centered at the origin of radius \( \frac{ab}{\sqrt{a^4 - b^4}} \).

Therefore, the points on the ellipse where the magnitude of the slope is 1 are given by this circle.

Trial 5, Thu, 19 Oct 2023

This try comes close, but ultimately fails. It commits the same magnitude=negation error that Trial 3 does and recovers similarly. Then it botches the final calculation of \( y \).

The slope of an ellipse at a point is given by:

\[
dy / dx = -b^2 x / a^2 y
\]

The magnitude of the slope is given by:

\[
|dy/dx| = b^2 x / a^2 y
\]
We want to find the coordinates of points where the magnitude of the slope is one. Therefore, we have:

\[ \frac{b^2 x}{a^2 y} = 1 \]

Solving for \( y \), we get:

\[ y = \frac{b^2 x}{a^2} \]

Substituting this value of \( y \) in the equation of the ellipse, we get:

\[ \frac{x^2}{a^2} + \frac{b^4 x^2}{(a^4 b^2)} = 1 \]

Simplifying, we get:

\[ (1 + \frac{b^2}{a^2}) x^2 = a^2 \]

Therefore, we have:

\[ x = \pm \frac{a}{\sqrt{1 + \frac{b^2}{a^2}}} \]

Substituting this value of \( x \) in the equation of the ellipse, we get:

\[ y = \pm \frac{b}{\sqrt{1 + \frac{b^2}{a^2}}} \]

Therefore, the coordinates of points where the magnitude of the slope of the ellipse is one are:

\[ (x, y) = (\pm \frac{a}{\sqrt{1 + \frac{b^2}{a^2}}}, \pm \frac{b}{\sqrt{1 + \frac{b^2}{a^2}}}) \]

If Chat were a person, I’d say the bad steps reveal carelessness; I expect better from a machine. The correct result is easy to find by symmetry. Because the problem is invariant under the simultaneous exchanges \( x \leftrightarrow y \) and \( a \leftrightarrow b \), we must have \( y = \pm b/\sqrt{1 + \frac{a^2}{b^2}} \). I prefer the more obviously symmetric (and corrected) form

\[ (x, y) = (\pm \frac{a}{\sqrt{a^2 + b^2}}, \pm \frac{b}{\sqrt{a^2 + b^2}}) \]

Trial 6, Fri, 20 Oct 2023

For this trial I replaced semiaxes \( a \) and \( b \) with \( p \) and \( q \). This didn’t phase Chat. Nevertheless, it got both simplification steps wrong. The presentation is otherwise clean. In particular, this is the only trial whose treatment of “magnitude” is fully correct.

The equation of the ellipse is:

\[ \frac{x^2}{p^2} + \frac{y^2}{q^2} = 1 \]

The slope of the ellipse at a point \( (x, y) \) is given by:

\[ \frac{dy}{dx} = -\frac{xq^2}{yp^2} \]

The magnitude of the slope is 1 when:

\[ \left| \frac{dy}{dx} \right| = 1 \]

Solving for \( y \) in terms of \( x \) and substituting in the above equation, we get:

\[ \left| -\frac{xq^2}{yp^2} \right| = 1 \]

which simplifies to:
\[ y = \pm \frac{q}{p} x \]

Substituting this value of \( y \) in the equation of the ellipse, we get:

\[ \frac{x^2}{p^2} + \frac{q^2 x^2}{p^2 q^2} = 1 \]

Simplifying, we get

\[ x = \pm \sqrt{\frac{p^2 q^2}{p^2 + q^2}} \]

Substituting this value of \( x \) in the equation of the line, we get:

\[ y = \pm \sqrt{\frac{p^2 q^2}{p^2 + q^2} \cdot \frac{q}{p}} \]

Therefore, the coordinates of the points where the magnitude of the slope of the ellipse is one are:

\[ \left( \sqrt{\frac{p^2 q^2}{p^2 + q^2}}, \sqrt{\frac{p^2 q^2}{p^2 + q^2} \cdot \frac{q}{p}} \right) \quad \text{and} \quad \left( -\sqrt{\frac{p^2 q^2}{p^2 + q^2}}, -\sqrt{\frac{p^2 q^2}{p^2 + q^2} \cdot \frac{q}{p}} \right) \]

Two more slip-ups: The magnitude is of the slope, not of the ellipse, and the \( \pm \) sign has vanished from the \( x \) coordinate.

Trial 7, Mon, 23 Oct 2023

This time the semi-axes were \( p \) and \( b \)—a typo on my part, but mathematically legitimate.

The equation of the ellipse is given by

\[ \frac{x^2}{p^2} + \frac{y^2}{b^2} = 1 \]

The slope of the ellipse at any point is given by

\[ -\frac{x(p^2/b^2)}{y} \]

Somehow the factor \( p^2/b^2 \) got inverted.

The magnitude of the slope is equal to 1 when \( |p^2/y| = |b^2/x| \)

Substituting \( y = (p^2/b^2)x \) in the equation of the ellipse, we get

\[ x = \sqrt{p^4/(b^4 + p^4)} \quad \text{and} \quad y = \sqrt{b^4/(b^4 + p^4)} \]

A bad substitution produces an answer in which \( x, y \) and \( p, b \) are measured in different units. Both formulas lack \( \pm \) signs.

Therefore, the coordinates of the points where the magnitude of the slope of the ellipse is one are

\[ \left( \sqrt{p^4/(b^4 + p^4)}, \sqrt{b^4/(b^4 + p^4)} \right) \quad \text{and} \quad \left( -\sqrt{p^4/(b^4 + p^4)}, -\sqrt{b^4/(b^4 + p^4)} \right) \]

Nothing is said about the origin of the negated \( x \) and \( y \) values or why only two solutions are given.

Trial 8, Sun, 5 Nov 2023

Two steps out of seven in this derivation come from nowhere. This seems to be about par for Chat.

The equation of the ellipse is given by
\[
\frac{x^2}{p^2} + \frac{y^2}{q^2} = 1
\]
The slope of the ellipse at any point is given by:
\[
\frac{dy}{dx} = -\frac{xq^2}{yp^2}
\]
The magnitude of the slope is given by:
\[
\left|\frac{dy}{dx}\right| = \left(\frac{q}{p}\right)\sqrt{\frac{p^2-x^2}{x^2}}
\]

Your name goes here if you can explain the origin of this formula.

To find the points where the magnitude of the slope is one, we need to solve the following equation:
\[
\frac{q}{p} \sqrt{\frac{p^2-x^2}{x^2}} = 1
\]

Squaring both sides, we get:
\[
\frac{p^2-x^2}{x^2} = \frac{p^2}{q^2}
\]

Simplifying, we get:
\[
x^2 = \frac{p^2q^2}{p^2+q^2}
\]

Therefore, the x-coordinates of the points where the magnitude of the slope is one are:
\[
x = \pm \sqrt{\frac{p^2q^2}{p^2+q^2}}
\]
The corresponding y-coordinates can be found by substituting these values of x into the equation of the ellipse:
\[
y = \pm \frac{qp}{\sqrt{p^2-x^2}}
\]

The denominator seems to have been plucked from the (bad) equation for the magnitude of the slope. The suggested substitution cannot have been made, for then x would not be present.

Therefore, the coordinates of the points where the magnitude of the slope of the ellipse is one are:
\[
\left(\sqrt{\frac{p^2q^2}{p^2+q^2}}, \pm \frac{qp}{\sqrt{p^2-x^2}}\right)
\]
and
\[
\left(-\sqrt{\frac{p^2q^2}{p^2+q^2}}, \pm \frac{qp}{\sqrt{p^2-x^2}}\right)
\]

Appendix. A correct solution

The equation of the ellipse is
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Differentiate with respect to \( x \).
\[ 2 \frac{x}{a^2} + 2y \frac{dy}{dx} \frac{1}{b^2} = 0 \]

Solve for the slope.
\[ \frac{dy}{dx} = -b^2 \frac{x}{a^2} y \]

The magnitude of the slope is 1 if and only if the square of the slope is 1.
\[ \left( \frac{dy}{dx} \right)^2 = b^4 \frac{x^2}{a^4} y^2 = 1 \]

Solve for \( y^2 \).
\[ y^2 = b^4 \frac{x^2}{a^4} \]

Substitute in the equation of the ellipse.
\[ \frac{x^2}{a^2} + b^4 \frac{x^2}{(a^4 b^2)} = 1 \]

Simplify.
\[ x^2 a^2 + x^2 b^2 = a^4 \]

Solve for \( x \).
\[ x = \pm a^2 / \sqrt{a^2 + b^2} \]

Substitute in the equation of the ellipse and solve for \( y \).
\[ y = \pm b^2 / \sqrt{a^2 + b^2} \]

For sanity checks, it is easy to see that these answers satisfy both the equation of the ellipse and the equation for the magnitude of the slope. They are dimensionally coherent, with \( x \) and \( y \) being measured in the same units (length) as \( a \) and \( b \). And they exhibit the same symmetry as the problem statement: invariance under simultaneous exchanges \( x \leftrightarrow y \) and \( a \leftrightarrow b \).

Doug McIlroy
5 Dec, 2023

**Baby math** added 29 Dec, 2023

Trivial tweaks 29 Dec, 2023 and 1 Jan, 2024