A Sensorless Insertion Strategy for Rigid Planar Parts

D. J. Balkcom *
Carnegie Mellon Robotics Institute
Pittsburgh, PA 15213
devin@ri.cmu.edu

E.J. Gottlieb and J.C. Trinkle †
Sandia National Laboratories
Albuquerque, NM 87185-1004
ejgottl@sandia.gov
jctrink@sandia.gov

Abstract

The companion paper [2] derives an algorithm that determines the external wrenches consistent with constraints on the contact interactions between two rigid planar bodies. In this paper, we show how this algorithm may be used to create sensorless plans that guarantee that a workpiece is correctly inserted into a fixture. Our method explicitly removes all wrenches consistent with undesirable contact modes, and therefore avoids the frictional indeterminacy problem.

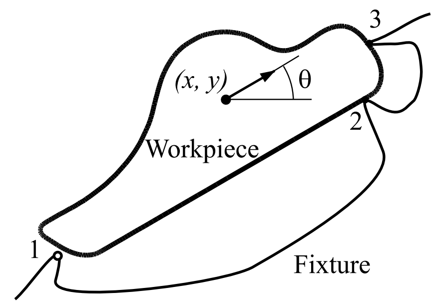

Figure 1: A workpiece nearly seated in a fixture.

1 Introduction

Most manufacturing and repair tasks require the use of clamps or fixtures. Fixtures are intended to hold a workpiece steady; the contact forces between the fixture and the workpiece balance any external wrench applied during the operation. Fixturing also guarantees that the workpiece is positioned correctly, if the shape of the fixture, the workpiece, and the intended contact locations are accurately known. We consider the problem of fixturing a planar workpiece.

The insertion process can be thought of as a finite state graph whose nodes represent the set of desired contacts that have been achieved. For example, if we call the desired contacts fixels 1, 2, and 3 (see figure 1), then one of the nodes of the transition graph will correspond to the case where contact has been achieved at fixels 2 and 3, but not at fixel 1. The goal state corresponds to the case where there is contact at all desired points.

The companion paper [2] presents an algorithm to determine the set of external wrenches consistent with given constraints on the relative accelerations and contact forces between a rigid planar workpiece and a fixture. In the companion paper, the algorithm is used to determine the set of external wrenches consistent with an initially motionless workpiece achieving a contact mode. In this paper, we describe how the algorithm may be applied to derive a set of external wrenches that guarantee that the workpiece will achieve all necessary contacts, regardless of the initial state. This set of wrenches can be mapped to the surface of the part to determine a region on the surface where pushing will seat the workpiece.

Relationship to previous work

Erdmann [4, 5] was among the first to use wrench cones to plan solutions to rigid body insertion tasks. Since then, wrench cones have been widely used in manipulation planning.

McCarragher's petri-net controller [1, 9] is an example of controller design for an insertion task. The contact state sensing was performed by comparing sensor signals to a qualitative template derived from rigid body dynamic equations under the frictionless assumption. Then the manipulator applied controls consistent with certain desirable state transitions, and not with other state transitions.

Unlike McCarragher's controller, passive (or sensor-

*The work of this author was supported by a Department of Energy Computational Sciences Graduate Fellowship.

†The work of these authors was supported by the Laboratory Directed Research and Development program of Sandia National Laboratories. Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.

less) controllers rely on a device designed to apply proper wrenches regardless of the current state. Erdmann and Mason develop a formal framework for the design of passive controllers in a paper on sensorless manipulation [6]. Whitney's remote center of compliance (RCC) [16] and Schimmels and Peshkin's accommodative wrist [13] are both examples of this approach. In the related problem of fixture clamp placement, Brost and Peters employed a quasistatic analysis of the clamping process over the range of motion of the clamp's plunger [3].

The method presented here falls into the category of passive insertion. One advantage of our method is ease of implementation: the algorithm for computing wrench cones presented in our companion paper [2] may be implemented in about one hundred lines of simple C code, and the method presented here only requires taking unions of some computed wrench cones. Like McCarragher, we assume that the state transitions are determined by a dynamic rather than a quasistatic rigid body model. We avoid the well-known nonuniqueness problem of rigid body dynamics (for example, see [14, 15]) by finding a set of external wrenches consistent *only* with seating the workpiece. Although we assume that workpiece velocities are small, our algorithm is also robust to sign changes in the tangential contact velocities.

2 Wrench cones consistent with contact modes

We assume that the workpiece is either touching or infinitessimally distant from each fixel, and that the workpiece velocity is small enough that velocity product terms may be neglected in the dynamic equations. We also assume Coulomb friction.

Under these assumptions, our companion paper [2] uses results from linear complementarity theory to derive an algorithm that determines the set of external wrenches consistent with a contact mode. The algorithm may be summarized as follows. Formulate the Newton-Euler equations, and choose a desired contact mode. Based on the contact mode, derive a set of constraints on the Newton-Euler equations that express kinematic non-penetration, Coulomb friction, and the observation that if a contact is breaking or has not been achieved, there will be no contact force. The algorithm transforms the Newton-Euler equations and the constraints into a simpler form that may be recognized as a polyhedral convex cone.

As an example, consider the disc-shaped workpiece shown in figure 2, motionless and touching two fixels. What external wrenches may cause contact to break at both fixels simultaneously?

If we want contact to break at both fixels, the workpiece must accelerate away from each fixel. Also, while the contacts are breaking, they cannot support a load. We define a_{in} and a_{it} to be the normal and tangential components of acceleration at fixel i, and define c_{in} and c_{it} to be the normal and tangential components of the contact force applied by fixel i. We collect the constraints:

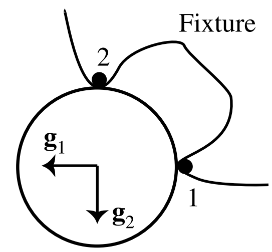

Figure 2: External wrenches that may cause contact to be broken at both fixels simultaneously.

$$a_{1\mathrm{n}} > 0,\ c_{1\mathrm{n}} = 0,\ c_{1\mathrm{t}} = 0 \tag{1}$$
$$a_{2\mathrm{n}} > 0,\ c_{2\mathrm{n}} = 0,\ c_{2\mathrm{t}} = 0 \tag{2}$$

$a_{1\mathrm{t}}$ and $a_{2\mathrm{t}}$ are unconstrained.

The input to the algorithm described in the companion paper is a set of constraints of the form described above, the location of the fixels, and the coefficient of friction. The output is a matrix $\mathbf{F}$, such that if the external wrench $\mathbf{g}$ is consistent is consistent with the constraints, then

$$\mathbf{F}\mathbf{g} \le 0 \tag{3}$$

For the example, it turns out that

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \mathbf{g} \le 0 \tag{4}$$

Figure 2 shows a geometric interpretation of inequality 4. Any external wrench $\mathbf{g}$ for which the force component is a positive linear combination of the two vectors $\mathbf{g}_1$ and $\mathbf{g}_2$ may cause separation from the two fixels simulaneously.

Constraints

As we saw in the simple example above, the interaction between the workpiece and the fixels constrains the contact accelerations and forces. We briefly enumerate the constraints corresponding to different contact interactions. For a more formal treatment of these constraints, the reader is referred to [11].

Non-penetration Assume there is contact at fixel i. Then $a_{in} \ge 0$; otherwise the fixel and the workpiece would interpenetrate.

Unilateral force Contact forces are unilateral: $c_{in} \geq 0$.

Coulomb friction We take the static and kinetic coefficients of friction to be μ_{s} and μ_{k}. If the workpiece is sliding to the right, then the frictional force will be on the left edge of the friction cone: $c_{it} = \mu_{\mathrm{k}} c_{in}$. If the workpiece is sliding to the left, then the frictional force will be on the right edge of the friction cone: $c_{it} = -\mu_{\mathrm{k}} c_{in}$. If the workpiece is not moving with respect to the fixture, then the friction force may fall anywhere in the static friction cone: $|c_{it}| \leq \mu_{\mathrm{s}} c_{in}$.

Separation If there is no contact between the workpiece and fixel i, or contact is breaking, fixel i cannot support a load and $c_{in} = c_{it} = 0$.

In the example, the contact mode mode (separating from both fixels) implied constraints on the contact forces and relative accelerations at each fixel. We describe the set of constraints by a string. The following table lists some possible constraints:

Abbrev.	a_{in}	a_{it}	c_{in}	c_{it}
s	≥ 0	-	0	0
a	< 0	-	0	0
u	-	-	0	0
l	0	> 0	≥ 0	$-\mu_{\mathrm{k}} c_{in}$
r	0	< 0	≥ 0	$\mu_{\mathrm{k}} c_{in}$
n	0	0	≥ 0	$\lvert c_{it}\rvert \leq \mu_{\mathrm{s}} c_{in}$
m	0	-	≥ 0	$\lvert c_{it}\rvert \leq \mu_{\mathrm{s}} c_{in}$

If the initial velocity of the workpiece is zero, then the constraints s, l, r, and n correspond to the usual definition of contact modes. For example, assume we have a workpiece that is contacting three fixels, and initially at rest. If we apply an external wrench consistent with the constraints lls, we expect the contact mode to be ‘sliding left over fixel 1, sliding left over fixel 2, and separating from fixel 3’ at the next time instant.

We will use $\mathcal{G}$ and a subscript to denote the set of external wrenches g satisfying a set of constraints. For example for the problem in figure 2, the notation $\mathcal{G}_{\mathsf{ss}}$ would be used describe the set of external wrenches consistent with the constraint that the workpiece separates from both fixels. The algorithm described in the companion paper can calculate a matrix describing the set $\mathcal{G}_{\mathsf{ss}}$ of external wrenches consistent with the impending contact mode.

For the purpose of planning insertion tasks, it is useful to define other constraints as well. Constraint a, ‘approaching’ is similar to s, but can only occur if there is no contact. Constraint u implies that there is no contact, but does not constrain the part to accelerate towards or away from the fixel in question. Finally, m, ‘maintain’, describes the situation where the normal acceleration is constrained so as to maintain contact, and the contact force is constrained to lie within the friction cone.

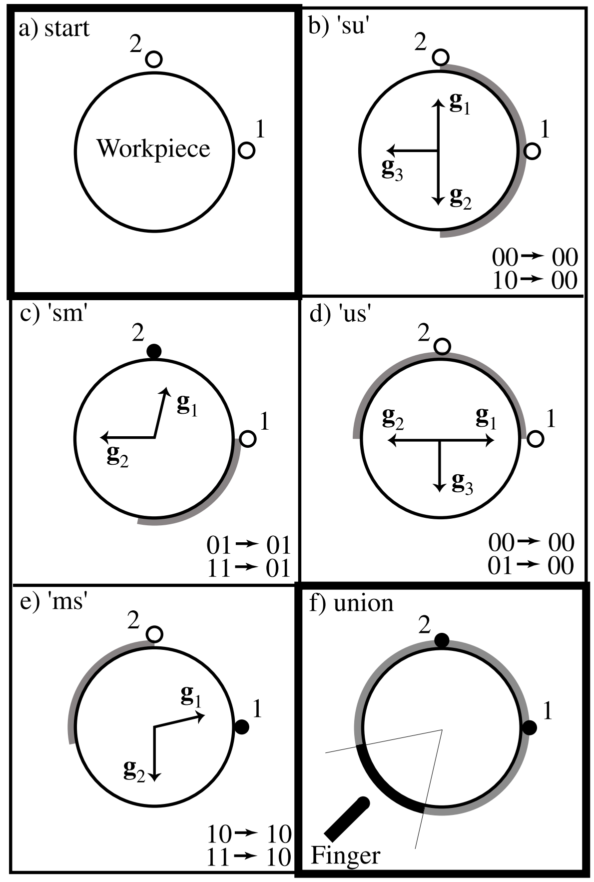

Figure 3: Finding the cone of wrenches that will cause a disc-shaped workpiece to contact two fixels, regardless of state.

Finally, we define the *contact state* to be the set of fixels at which contact has been achieved, and describe the current state by a binary number. State 00 describes the case where no contact had been achieved, and state 10 describes the case where contact had been achieved at the first fixel but not at the second.

3 Example problem, two contacts

Consider figure 3a. The problem is to seat a disc-shaped workpiece of uniform density against the two fixels. Figure 4a shows the possible contact states and state transitions. We assume that the workpiece is quite close to both fixels initially and impact can be ignored; if the workpiece accelerates towards a fixel, it will eventually achieve contact and the normal velocity will reach zero.

By choosing constraints on contact forces and accelerations we may enable or disable state transitions. Consider figure 4b. If the constraint au is satified and there is no

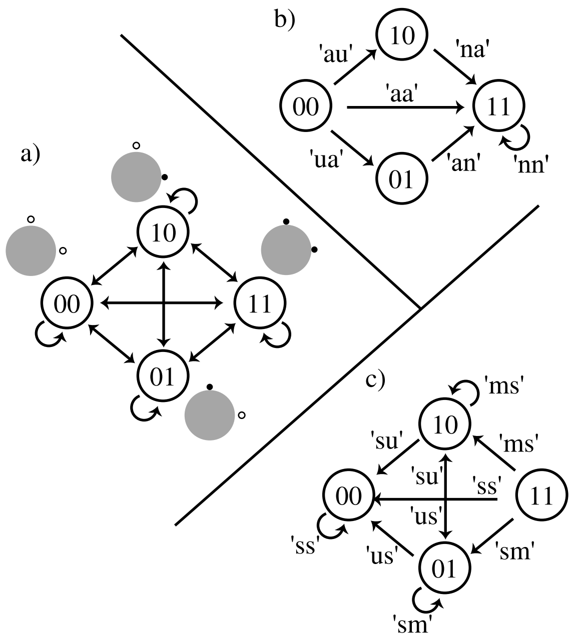

Figure 4: State transition graphs, two fixels.

contact, then the state transition 00 $\rightarrow$ 10 is likely, since the constraint au ensures that the workpiece will accelerate towards the first fixel. Similarly, the constraints ua, aa, na, and an are all desirable, since they are consistent with state transitions that bring the system closer to the goal state.

It might seem that choosing an external wrench from the set

$$\mathcal{G}_{\mathsf{au}} \cap \mathcal{G}_{\mathsf{ua}} \cap \mathcal{G}_{\mathsf{aa}} \cap \mathcal{G}_{\mathsf{na}} \cap \mathcal{G}_{\mathsf{an}} \cap \mathcal{G}_{\mathsf{nn}} \tag{5}$$

would guarantee that the goal would be reached. However, the algorithm presented in [2] only finds the set of external wrenches *consistent* with constraints on the contact forces and accelerations. Due to the well-known rigid body non-uniqueness problem (see [7] for example), it is possible that a single external wrench may be consistent with more than one vector of contact forces and accelerations.

Therefore, instead of attempting to enable forwards transitions, we disable the backwards and self transitions by ensuring that the external wrench is not consistent with the constraints shown in figure 4c. Consider the state 00, where no contact has been achieved. If we apply a wrench *not* in the set $\mathcal{G}_{\mathsf{su}}$, then the workpiece will accelerate towards the first fixel, and we expect the workpiece to eventually contact either fixel 1 or fixel 2. Analysis of the remaining state transitions is similar. If we choose an external wrench from the set

$$\mathcal{G}_{\text{goal}} = \overline{\mathcal{G}_{\mathsf{su}} \cup \mathcal{G}_{\mathsf{us}} \cup \mathcal{G}_{\mathsf{sm}} \cup \mathcal{G}_{\mathsf{ms}}} \tag{6}$$

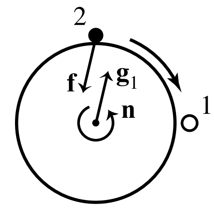

Figure 5: Spinning disc example.

then the number of contacts achieved will monotonically increase until the goal is achieved, regardless of the initial state.

Figures 3b, 3c, 3d, and 3e show the external wrenches consistent with each of the backwards and self transitions. Since the workpiece is a disc, arbitrary torques may be applied about the center without causing the workpiece to approach or separate from the fixels. The consistent external forces are positive linear combinations of forces acting along the vectors shown. The thick gray lines show places on the surface of the part where pushing with a frictionless finger would generate these forces. Figure 3f shows the union of these undesirable places to push (gray), and the complement of the union (black). Pushing on the black region will seat the workpiece, regardless of initial state.

It may seem surprising that we consider wrench sets involving the constraint m, rather than the constraints l, r, and n, corresponding to contact modes. The constraint m implies that the normal component of acceleration be zero for the fixel in question, but does not constrain the direction of the tangential acceleration or of the tangential contact force. Therefore, for a single fixel,

$$\mathcal{G}_{\mathsf{m}} \supset \mathcal{G}_{\mathsf{l}} \cup \mathcal{G}_{\mathsf{r}} \cup \mathcal{G}_{\mathsf{n}} \tag{7}$$

Calculating $\mathcal{G}_{\mathsf{sm}}$ and $\mathcal{G}_{\mathsf{ms}}$ rather than $\mathcal{G}_{\mathsf{sl}}$, $\mathcal{G}_{\mathsf{sr}}$, $\mathcal{G}_{\mathsf{sn}}$, $\mathcal{G}_{\mathsf{ls}}$, $\mathcal{G}_{\mathsf{rs}}$, and $\mathcal{G}_{\mathsf{ns}}$ saves some computation. There is an additional advantage: the constraint m makes no assumption about the direction of the tangential velocity. Consider figure 3c. It might seem that a force applied along $\mathbf{g}_1$ would ensure that contact would be made at fixels 1 and 2. And in fact,

$$\mathbf{g}_1 \notin \mathcal{G}_{\mathsf{sl}} \cup \mathcal{G}_{\mathsf{sr}} \cup \mathcal{G}_{\mathsf{sn}} \tag{8}$$

Why then is $\mathbf{g}_1$ excluded from $\mathcal{G}_{\text{goal}}$? The difficulty is that the constraints l, r, and n are defined in terms of accelerations, while the frictional contact force depend on velocities. If there are non-zero velocities, then there is a case where applying a force $\mathbf{g}_1$ will not cause acceleration of the workpiece towards fixel 1. Figure 5 shows an example, constructed from figure 3c. The workpiece is initially spinning clockwise slowly. Therefore the force applied by fixel 2 is along an edge of the friction cone, labelled $\mathbf{f}$ in

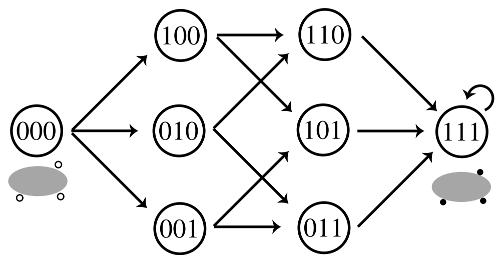

Figure 6: State transition graph, three fixels

the figure. If we apply a force along $\mathbf{g}_1$, the total wrench may be just a counter-clockwise torque $\mathbf{n}$ around the center of the disc; the workpiece will not accelerate towards fixel 1. Since calculating the sets $\mathcal{G}_{\mathsf{sm}}$ and $\mathcal{G}_{\mathsf{ms}}$ does not require an assumption about what side of the friction cone the contact force will lie on, this case is correctly handled and the sensorless plan described above is robust to sign changes in the tangential contact velocities.

4 Three fixels

A similar strategy may be applied if three contacts are to be achieved. Figure 6 shows a state transition graph that guarantees that the workpiece will be seated. (Our approach is conservative, since this is not the only graph that guarantees that the goal will be reached.) In order to ensure that the wrench applied is consistent only with this state transition graph and with no others, we consider all possible state transitions. We summarize the results in tabular form:

State	Constraints
000	sss
100	**suu**, mss
010	**usu**, sms
001	**uus**, ssm
110	ssu, **msu**, **smu**, **mms**
101	sus, **mus**, **sum**, **msm**
011	uss, **ums**, **usm**, **smm**
111	sss, mss, sms, mss, mms, smm, msm

Some of the constraints listed are redundant. For example, if a wrench is in the set $\overline{\mathcal{G}_{\mathsf{suu}}}$, then it is also in the set $\overline{\mathcal{G}_{\mathsf{sss}}}$. In all, twelve sets must be computed, corresponding to the constraints shown in bold. Wrenches in the complement of the union of these sets ensure that the goal will be reached. Figure 7 shows an example generated by our sample implementation of the algorithm. Pushing along the thick black curve with a frictionless finger will seat the workpiece, regardless of initial state.

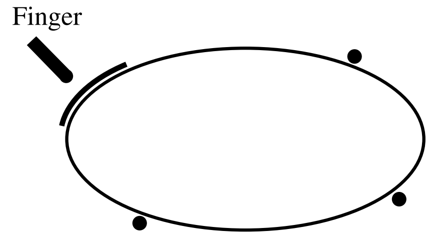

Figure 7: Seating a workpiece against three fixels.

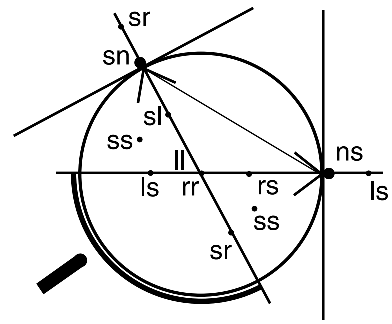

Figure 8: Computing strong stability

5 Strong stability at the goal

Once the workpiece has contacted all desired fixels, it is important to know which external wrenches may cause the workpiece to move. Pang and Trinkle [11] make the following definitions:

Weak Stability: There exists a solution to the rigid body dynamics model for which the acceleration of the workpiece is zero.

Strong Stability: The acceleration of the workpiece is zero for all solutions of the rigid body dynamics model.

If we want the part to be motionless at the goal, it is necessary that any applied external wrench be in the *weakly stable cone*. It is sufficient that the external wrench be in the *strongly stable cone*.

The weak stability problem has been well studied. Take the set of unit wrenches corresponding to the edges of the friction cones; these wrenches are the edges of a polyhedral convex cone in wrench space. If the negative of the external wrench is included in this cone, then the part is weakly stable with respect to that wrench. Weak stability corresponds to the constraints nn (for two fixels) and nnn for three fixels. In figure 8, μ_s is large enough that each fixel is included in the friction cone of the other. Nguyen's condition for force closure is satisfied ([10]), and thus it is *possible* that any external wrench will be balanced by the contact forces. The weakly stable cone is all of wrench space.

In order to *guarantee* that the workpiece will not move, we may calculate the strong stability cone by the following method:

1. Determine the kinematically feasible contact modes. (See Mason chapter 8 [8] for an algorithm based on Reuleaux's method [12].)
2. For each contact mode except n...n calculate the set of consistent wrenches. Take the complement of the union.

The workpiece is then strongly stable with respect to the calculated wrenches. Figure 8 shows an example. The kinematically feasible contact modes are ss, sl, sr, sn, ls, rs, ns, ll, and rr. We calculate

$$\mathcal{G}_{\text{strong}} = \overline{\mathcal{G}_{\mathsf{ss}} \cup \mathcal{G}_{\mathsf{sl}} \cup \mathcal{G}_{\mathsf{sr}\cdots}} \tag{9}$$

Figure 8 shows the places where pushing with a frictionless finger would exert an external wrench in the strongly stable cone. The workpiece will not move if the finger pushes along the dark line.

Computation of the strongly stable cone allows an analysis of situations where immobilization is to be achieved by a combination of external forces and geometric constraints. If $\mathcal{G}_{\text{strong}}$ has non-zero volume, and we apply a biasing wrench from the interior of $\mathcal{G}_{\text{strong}}$, then the workpiece will not move under small disturbance wrenches of arbitrary direction.

Since $\mathcal{G}_{\text{goal}} \subset \mathcal{G}_{\text{strong}}$, a wrench from the interior of $\mathcal{G}_{\text{goal}}$ can act as the biasing wrench. If the workpiece is seated by gravity, then disturbance wrenches of less than a certain magnitude will not move the workpiece after it has been seated as long as the gravitational wrench remains the same. The magnitude of the permitted disturbance forces may be calculated from the shape of the strongly stable cone and the biasing wrench applied.

6 Conclusion and Acknowledgments

We presented a method to determine a set of external wrenches consistent only with state transitions that increase the number of contacts. This method explicitly avoids the problem of non-uniqueness of solutions to the rigid body dynamics problems with Coulomb friction. Additionally, we showed how to compute the cone of external wrenches with respect to which two contacting rigid bodies are *strongly stable*. The wrench set derived to seat the workpiece is a subset of the strongly stable cone.

The authors wish to acknowledge the helpful comments and insights provided by Mathew T. Mason and by Jong-Shi Pang.

References

[1] D. Austin and B. McCarragher. Robust discrete event controller synthesis for constrained motion systems. In *Symposium on Robot Control*, pages 801–807. Elsevier, September 1997.

[2] D. J. Balkcom, J. Trinkle, and E. Gottlieb. Computing wrench cones for planar contact tasks. In *Proceedings, IEEE International Conference on Robotics and Automation*, 2002.

[3] R. Brost and R. Peters. Automatic design of 3d fixtures and assembly pallets. Technical Report SAND95-2411, Sandia National Laboratories, January 1997.

[4] M. A. Erdmann. On motion planning with uncertainty. Master's thesis, MIT Department of Electrical Engineering and Computer Science, August 1984.

[5] M. A. Erdmann. On a representation of friction in configuration space. *International Journal of Robotics Research*, 13(3):240–271, June 1994.

[6] M. A. Erdmann and M. T. Mason. An exploration of sensorless manipulation. *IEEE Journal of Robotics and Automation*, 4(4):369–379, August 1988.

[7] P. Lötstedt. Mechanical systems of rigid bodies subject to unilateral constraints. *SIAM Journal of Applied Mathematics*, 42(2):281–296, 1982.

[8] M. T. Mason. *Mechanics of robotic manipulation*. MIT Press, 2001.

[9] B. McCarragher and H. Asada. A discrete event controller using petri nets applied to assembly. In *Proceedings, IEEE International Conference on Intelligent Robots and Systems*, pages 2087–2094, 1992.

[10] V. Nguyen. Constructing force-closure grasps. *International Journal of Robotics Research*, 7(3):3–16, June 1988.

[11] J. Pang and J. Trinkle. Stability characterizations of rigid body contact problems with coulomb friction. *Zeitschrift für Angewandte Mathematik und Mechanik*, 80(10):643–663, 2000.

[12] F. Reuleaux. *The Kinematics of Machinery*. Macmillan, 1876. Republished by Dover, New York, 1963.

[13] J. M. Schimmels and M. A. Peshkin. Force assembly with friction. *IEEE Transactions on Robotics and Automation*, 10(4):465–497, 1994.

[14] J. Trinkle, J. Pang, S. Sudarsky, and G. Lo. On dynamic multi-rigid-body contact problems with coulomb friction. *Zeitschrift für Angewandte Mathematik und Mechanik*, 77(4):267–279, 1997.

[15] J. Trinkle and D. Zeng. Prediction of the quasistatic planar motion of a contacted rigid body. *IEEE Transactions on Robotics and Automation*, 11(2):229–246, April 1995.

[16] D. E. Whitney. Quasi-static assembly of compliantly supported rigid parts. *Journal of Dynamic Systems, Measurement, and Control*, 104:65–77, March 1982.