Noise & Denoising

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Today’s agenda

Info on paper discussions

Noise

Denoising with multiple images

- Probability review

Noise characteristics

Sources of noise

Class portraits!
Presentation guidelines

30 minutes per paper (presentation + discussion)

You may use Powerpoint/Keynote, blackboard, etc.

Focus on getting across the main points of the paper first

- Present the paper as if everyone skimmed it but forgot it, or didn't understand.

- First present the problem that the paper solves and the general approach.

- You should then give a clear and concise description of the main technical parts of the paper (algorithms, equations, etc).
Presentation guidelines

Everyone will have read the paper before class.

- Your job should not be simply reciting what is in the paper
- Go beyond that, working out exactly how the algorithm (or theory) works and deciding how to present this in class.
- The best way to present an approach may not be the order in which things are described in the paper.
Leave no stone unturned

A paper's content may not be sufficient to fully describe how a technique works.

- may depend on prior papers/techniques

A major goal of your presentation is to fill in these gaps and present a complete picture of the paper in class.

If there is something you don't understand, you must either work it out yourself, or come to office hours so that we can resolve it together.
Presentation guidelines

Some authors provide presentations and other material online.

- Proper attribution rules apply

Practice, practice, practice

- You should practice your presentation at home, and time yourself, before coming to class.

- Pay attention to what you did (and did not) like about your classmates’ presentation style, level of preparation, etc. with an eye toward improving your own presentation skills.
Paper discussion

Everyone else will not be a passive observer

Discussant will initiate and facilitate the discussion

Everyone is expected to participate in the discussions
Things to think about

Limitations
- Do you think everything will work as described?
- What are the corner/failure cases?
- The paper may not be forthcoming about limitations

Future work?

Relations/comparisons
- How does the paper relate to other papers we have read?
- Can you imagine applying the ideas to a different problem?
Noise
Noisy image

Usually for dark conditions
Noise

Fluctuation when taking multiple shots
Canon 1D mark II N at ISO 3200

What should the histogram be within this box?
Photoshop demo
Histogram of grey patch

Should be single values for RGB (constant color)
Recap

Noise exists

Noise can be observed as:
- fluctuation over time
- fluctuation over space when should be constant
Denoising by averaging
Averaging pseudo-code

```python
mean = imSeq[1]
for i = 2 to imSeq.size():
    mean += imSeq[i]
mean /= imSeq.size()
```
3 images
5 images
Probabilistic perspective
Noise statistics / probability

Denote pixel values like random variables $X$

Mean: $\mu$ or $E[x]$, the true measurement

Variance: $\sigma^2[x] = \sim$ average squared error
- more precisely: average squared difference to mean

$$\sigma^2[x] = E[(E[x] - x)^2]$$

$$\sigma^2 = E[x^2] - E[x]^2$$

Standard deviation: $\sigma[x] = \text{square root of variance}$
- In same unit as measurement
Estimating the sample mean

Say we have N measurements $x_i$

How would you estimate their mean?

$$\mu_N = \frac{1}{N} \sum x_i$$
Estimating the sample variance

Say we have \( N \) measurements \( x_i \)

How would you estimate their variance?

Use original definition:

\[
\sigma^2[x] = E[(E[x] - x)^2]
\]

\[
\mu_N = \frac{1}{N} \sum x_i \quad \sigma_N^2 = \frac{1}{N} \sum (\mu_N - x_i)^2
\]

- this underestimates variance!
Estimating the sample variance

\[
\sigma^2_N = \frac{1}{N} \sum (\mu_N - x_i)^2 \quad \sigma^2_N = \frac{1}{N - 1} \sum (\mu_N - x_i)^2
\]

Divide by N-1, not by N

- Otherwise, variance would be underestimated on average
- called Bessel correction: removes bias
- Intuition: we use the same samples for estimating the mean and variance, which introduces correlation that underestimates variance
Example with coin flip, N=2

We do 2 coin flips

Try to estimate mean & variance

Sometimes we’ll be wrong

- e.g. if we get 0 twice, we’ll think variance is zero
- but we’d like to be right on average (called unbiased)
Example with coin flip, N=2

4 scenarios: (0,0) ; (0, 1) ; (1, 0) ; (1, 1)
- mean estimates: 0 ; 0.5 ; 0.5 ; 1
- average of the mean estimations: 0.5, equal to true mean (unbiased)

true variance: 0.25

sum of squared differences to sample mean: 0; 0.5; 0.5; 0

estimator $\sigma^2_N = \frac{1}{N} \sum (\mu_N - x_i)^2$ 0 ; 0.25 ; 0.25 ; 0
- 0.125 on average, biased

estimator $\sigma^2_N = \frac{1}{N - 1} \sum (\mu_N - x_i)^2$ 0 ; 0.5 ; 0.5 ; 0
- 0.25 on average, unbiased

After a slide by Frédo Durand
Signal-to-noise ratio (SNR)

\[
SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma}
\]

\[
\log{SNR(\text{dB})} = 10 \log_{10} \left( \frac{\mu^2}{\sigma^2} \right) = 20 \log_{10} \left( \frac{\mu}{\sigma} \right)
\]
SNR in practice

Be careful. Sometimes variance is zero (for no good reason) and will break things

- practical hack: take the max of $\sigma^2$ and a small number, e.g. 1e-6
Basic probability tools
Goal

Analyze how the mean & variance evolve when we denoise by averaging multiple frames

Formula for average: \( \frac{1}{N} \sum x_i \)

- addition
- multiply by scalar
Expected value

\[ E[kx] = \]
\[ E[x+y] = \]
\[ E[xy] = \]
Expected value

\[ E[kx] = kE[x] \]
\[ E[x+y] = \]
\[ E[xy] = \]
Expected value

\[ E[kx] = kE[x] \]

\[ E[x+y] = E[x] + E[y] \]

\[ E[xy] = \]
Expected value

\[ E[kx] = kE[x] \]

\[ E[x+y] = E[x]+E[y] \]

\[ E[xy] = E[x]E[y]? \]
Expected value

\[ E[kx] = kE[x] \]
\[ E[x+y] = E[x] + E[y] \]
\[ E[xy] = E[x]E[y] \]

- only when they are uncorrelated!

\[ E[xy] = \int \int xy \ p(x, y) \ dx \ dy \]
\[ = \int \int xy \ p(x)p(y) \ dx \ dy \]
\[ = \int p(y) \ y \int x \ p(x) \ dx \ dy \]
\[ = \int p(y) \ yE[x] \ dy \]
\[ = E[y]E[x] \]
Variance identity

\[
\sigma^2[x] = E[(E[x] - x)^2]
\]

\[
= E[E[x]^2 - 2xE[x] + x^2]
\]

\[
= E[x]^2 - 2E[x]E[x] + E[x^2]
\]

\[
= -E[x]^2 + E[x^2]
\]

\[
\sigma^2 = E[x^2] - E[x]^2
\]
Variance properties

Multiplication by $k$:

$$\sigma^2[kx] = E[(kx)^2] - E[kx]^2 = k^2 \sigma^2[x]$$

not linear, quadratic!

Addition of two random variables

$$\sigma^2[x + y] = E[(x + y)^2] - E[x + y]^2$$

$$= E[x^2 + 2xy + y^2] - (E[x] + E[y])^2$$


uncorrelated: $E[xy]=E[x]E[y]$

$$= E[x^2] - E[x]^2 + E[y^2] - E[y]^2$$

$$= \sigma^2[x] + \sigma^2[y]$$

variance is additive!
Take home message

Noise/measurement as random variable

Mean, variance, standard deviation

Variance:

- multiplication by $k \Rightarrow k^2$
- addition $\Rightarrow$ addition

SNR, log of SNR
Convergence
Convergence

Assume images are IID random measurements

Variance for one image: $\sigma^2[x_i]$

Average: $\frac{1}{N} \sum_{i=1}^{N} x_i$

What is the variance of the average?
Convergence

Assume images are IID random measurements

Variance for one image: \( \sigma^2[x_i] \)

Average:

\[
\frac{1}{N} \sum_{i=1}^{N} x_i
\]

\[
\sigma^2 \left[ \frac{1}{N} \sum_{i=1}^{N} x_i \right] = \left( \frac{1}{N} \right)^2 \sum_{i=1}^{N} \sigma^2[x_i]
\]

\[
= \left( \frac{1}{N} \right)^2 N \sigma^2[x_i]
\]

\[
= \frac{1}{N} \sigma^2[x_i]
\]
IMPORTANT RESULT

Denoising by averaging:

- variance is reduced as 1/N

- standard deviation (error) is reduced by sqrt(N)
Alignment
Brute force

Assignment 3!

Try all possible shifts within +/- maxOffset

Keep the one with minimum sum of square differences
Casio EXF1, Google glass

Can do denoising by aligning and averaging N images
Noise characteristics
Analyzing noise

Camera on tripod, many pictures

Compute mean, variance, stddev, SNR
Exposure

Get the right amount of light to sensor/film

Two main parameters:

- Shutter speed
- Aperture (area of lens)
+ sensor sensitivity (ISO)

In what follows, I kept the exposure the same and explored the tradeoff between shutter speed and ISO
Canon 1D II N at ISO 3200

After a slide by Frédéric Durand
Canon 1D II N at ISO 3200

Looks noisy, especially in dark areas
Canon 1D II N at ISO 3200

Denoised with 45 images (estimator of mean)
Canon 1D II N at ISO 3200

Standard deviation (some alignment issues...)

After a slide by Frédou Durand
For each pixel, for each channel, compute

\[
\sigma_N = \sqrt{\frac{1}{N-1} \sum (\mu_N - x_i)^2}
\]

and display as an image
Canon 1D II N at ISO 3200

Standard deviation (some alignment issues...)

Observations:
more noise in bright image areas

more  less

After a slide by Frédo Durand
Canon 1D II N at ISO 3200

log SNR
Canon 1D II at ISO 3200

log SNR – looks a lot like the image!

even though we have more noise, bright areas have better SNR
Observations

Noise is more visible in dark areas
Noise is numerically higher in bright areas
SNR is better in bright areas
Canon 1D II N at ISO 3200
Canon 1D II, ISO 100

A lot less noisy!
Canon 1D IIN at ISO 3200

Standard deviation (some alignment issues...)

After a slide by Frédo Durand
Canon 1D II at ISO 100

Standard deviation (some alignment issues...)
Canon 1D II N at ISO 3200

log SNR
Canon 1D II N at ISO 1600

log SNR
Canon 1D II N at ISO 400

log SNR
Nikon D3s at 1600 ISO

After a slide by Frédo Durand
Nikon D3s at 1600 ISO
Canon 1D Mark II N at 1600 ISO

After a slide by Frédéric Durand
Recap and questions?

Noise level depends on

- pixel intensity
- ISO
- color channel
- camera
Sources of noise
Photon shot noise

The number of photons arriving during an exposure varies from exposure to exposure and from pixel to pixel, even if the scene is completely uniform.

On average you might get 100 photons, but sometimes it will be 98, sometimes 103, etc.

This phenomenon is governed by physics and the value follows the Poisson distribution.
Poisson distribution

Expresses the probability that a certain number of events will occur during an interval of time

Applicable to events that occur

- with a known average rate, and
- independently of the time since the last event

If on average \( \lambda \) events occur in an interval of time, the probability \( p \) that \( k \) events occur instead is

\[
p(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}
\]
Poisson distribution

The mean and variance of the Poisson distribution are

\[ \mu = \lambda \quad \text{and} \quad \sigma^2 = \lambda \]

The standard deviation is

\[ \sigma = \sqrt{\lambda} \]

Deviation grows slower than average.

\[ p(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \]
Photon shot noise

Photons arrive in a Poisson distribution

\[ \mu = \lambda \quad \sigma = \sqrt{\lambda} \]

so

\[ SNR = \frac{\mu}{\sigma} = \sqrt{\lambda} \]

Shot noise scales as square root of number of photons
Canon 1D IIN at ISO 3200

log SNR – dominated by Poisson, $\sim\sqrt{\text{image}}$
Dark current

Electrons dislodged by random thermal activity

Increases linearly with exposure time

Increases exponentially with temperature

Varies across sensor

(http://theory.uchicago.edu/~ejm/pix/20d/tests/noise/)

Canon 20D, 612 sec exposure
Hot pixels

Electrons leaking into well due to manufacturing defects

Increases linearly with exposure time

Increases with temperature, but hard to model

Changes over time, and every camera has them
Fixing dark current and hot pixels

Solution #1: chill the sensor

Solution #2: dark frame subtraction
- available on high-end SLRs
- compensates for average dark current
- also compensates for hot pixels and FPN
Fixed pattern noise (FPN)

Manufacturing variations across pixels, columns, blocks

Mainly in CMOS sensors

Doesn’t change over time, so read once and subtract

Canon 20D, ISO 800, cropped
Read noise

Thermal noise in readout circuitry

Again, mainly in CMOS sensors

Not fixed patterns, so only solution is cooling

Canon 1Ds Mark III, cropped
Recap

Photon shot noise
- **unavoidable** randomness in number of photons arriving
- grows as the sqrt of # photons, so brighter lighting and longer exposures will be less noisy

Dark current noise
- grows with exposure time and sensor temperature
- minimal for most exposure times used in photography
- correct by subtraction, but only corrects for average dark current

Hot pixels, fixed pattern noise
- caused by manufacturing defects, correct by subtraction

Read noise
- electronic noise when reading pixels, **unavoidable**
Digital pipeline

Photosites transform photons into charge (electrons)
- The sensor itself is linear

Gets amplified (depending on ISO setting)

Then goes through analog-to-digital converter
- up to 14 bits/channel these days

*Stop here when shooting RAW*

Then demosaicing, denoising, white balance, a response curve, gamma encoding are applied

Quantized and recorded as 8-bit JPEG
Pipeline & noise

This is a conceptual diagram, don’t take it too literally

- e.g. the A-to-D converter is a serious source of noise, but usually electronic noise, not quantization artifacts

Noise = (photon noise + readout noise) * amplification + post-amplification noise
ISO amplifies

e.g. going from ISO 100 to ISO 400 amplifies by 4 both noise & signal
ISO 100

A lot less noisy!
Pipeline & noise

For a given signal level, and a given desired image brightness...

Two alternatives
- use low ISO and brighten digitally
- use high ISO to get brightness directly

The latter gives less noise because you don’t amplify post-gain noise
ISO recap

ISO is a simple gain
- amplifies noise as well

But when the signal is low, it’s better to amplify as early as possible (ISO rather than digitally)

Ideally, you make sure the signal is high by use a slower exposure or larger aperture
Brain teaser

For the same light level and electronic (per photosite read noise), and same total sensor size, is it better to:

- have a 16 Mpixel sensor and average groups of 4 pixels to yield a 4 Mpixel image
- have a 4Mpixel sensor (with bigger photosites)

Analyze photon noise and read noise
- careful about adding vs. averaging
Different regimes

For bright pixels (in fact, most pixels), photon noise dominates

For dark pixels: electronic (read) noise dominates

For long exposures, thermal noise kicks in
Questions?
Slide credits

Frédo Durand
Marc Levoy