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Chapter 1

Market Driven Sharing of Spectrum in Infrastructure Networks

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1.1 Introduction

Access to radio spectrum is a key requirement for continuous wireless growth and deployment of new mobile services. Given the fast growing demand for radio spectrum, regulators around the world are implementing much more flexible and liberal forms of spectrum management, often referred to as dynamic spectrum management. This new models dynamically redistribute and re-assign spectrum with each and across different wireless systems, adapting spectrum usage to actual demands and achieving much more efficient use of the precious spectrum resource. Within the new model, two prominent approaches that are being considered by the regulators are spectrum trading and cognitive spectrum access [1]. In this chapter, we focus on examining challenges and solutions in the area of spectrum trading.

Spectrum trading is a market-based approach for spectrum redistribution which enables a spectrum licence holder (for example a cellular operator) to sell or lease all or a portion of its spectrum to a third party. The third party can then in principle change the use of spectrum or the technology to be used provided that certain conditions are satisfied. Note that this is an important departure from the Command & Control management model, where spectrum licences were granted by regulators for the provision of a specific service using a pre-defined technology,
and licence holders were not allowed to reallocate their spectrum to different technologies or other users. Exposing the radio spectrum to market forces has become increasingly popular. For example, the UK regulator, Ofcom, is aiming that by 2010 71.5% of its available spectrum should be operating under market forces [2] (see Figure 1.1). The rationale for the approach is that market mechanism will allocate spectrum to those who value it most, thereby ensuring that the (economically) most efficient utilization of this resource is achieved. However, at least initially, one expects that such forms of spectrum trading would only take place on a macro-scale (e.g., between two cellular service providers) involving large blocks of spectrum and timescales that are still dictated by complex and cumbersome bureaucratic procedures involved in such wholesale forms of trading.

**Dynamic Spectrum Micro-Auctions.** While cognitive access to certain "publicly owned" licensed bands, such as TV and military bands, are being actively pursued by regulators, it is very doubtful that without any economic incentive this form of access can be extended to "privately owned" licensed bands, such as 3G spectrum, for which the incumbents have already paid billions of dollars/pounds/Euros in order to ensure their exclusive use. It is, therefore, clear that in order to make possible secondary access by cognitive radios to such licensed bands market mechanisms on a micro-scale need to be implemented in order to create economic incentives for licensed holder for sharing their spectrum locally and temporarily with cognitive radios.

In order for market players (cognitive radios and incumbent systems) to make economically efficient deals, they require a market environment that enables them to negotiate such that mutually acceptable bargains are reached. Auctions are among the best-known market-based allocation mechanisms due to their perceived fairness and allocation efficiency. Indeed, FCC (Federal Communications Com-
mission) and its counterparts across the world have extensively used auctions for wholesale allocation of spectrum in the last decade and intend to use this mechanism in the future. However, a FCC-style spectrum auction targets long-term national/regional leases, requiring huge up-front investments. In this chapter, on the other hand, our focus is on micro-auction mechanism that allow for the trading of spectrum rights at network level. These types of auction mechanisms could be highly attractive to network operators, as they provide a flexible and cost-effective means for dynamic expansion of their spectrum resources without the need for costly capital investments in new spectrum. The spectrum obtained through micro-auctions can be used for congestion relief during peak loads in traffic, or to enhance existing services and provide new services without the need for acquiring additional spectrum. More generally, users will be able to dynamically and locally vary their operating frequencies and access the best available spectrum on a "just-in-time" basis. This may happen either upon instruction from a cognitive base station that acquires spectrum on behalf of users [3], or autonomously by user devices themselves.

The Role of Cognitive Radios. Cognitive functionality is essential in realization of such types of micro-auctions because wireless devices can understand the regulatory, technical and economic context within which they found themselves and be able to perform the required negotiation and decision making task that are involved in the bidding procedure in such auctions. The scope of this chapter, however, is not on developing such cognitive functionalities. Instead we shall assume that these functionalities will be available in future devices and focus on developing and modeling appropriate auction algorithms which ensure fast and efficient redistribution of spectrum on network level. Furthermore, we will not have any assumption regarding the underlying network access technologies that a cognitive device uses for its transmissions once it acquires a portion of spectrum. However, following [4], we envisage that access technologies such as OFDMA will play an important role in enabling our micro-auction mechanisms. These technologies will support dynamic bandwidth availability and permit grouping, sub-dividing, and pooling of pieces of spectrum into neatly packaged spectrum channels.

1.2 Rethinking Spectrum Auctions

In the past decade, radio spectrum has always been auctioned in terms of pre-partitioned bulk licenses that cannot match time-varying market demands. Such mismatch has led to several negative consequences. First, forced to bid in the unit of bulk licenses, buyers face huge up-front costs. As a result, past auctions involved only a very few large (incumber) players, required significant manual negotiations and often took months or years to conclude. Second, winning buyers who received
the licenses cannot efficiently utilize assigned spectrum because their traffic varies significantly in time and space. Finally, while winning buyers’ spectrum sits unused, new entrants and new wireless technologies are either blocked or forced to crowd into highly unreliable unlicensed bands. If not addressed, such inefficiency will soon put a stop to wireless growth and innovation.

Solving such inefficiency requires us to rethink the way spectrum is distributed, and to redesign spectrum auctions to provide networks with spectrum matching their individual demands. Recent works have proposed an eBay-like, open marketplace concept to enable dynamic spectrum trading [5, 6]. In this marketplace, existing spectrum owners (as providers) gain financial returns by leasing their idle spectrum to new spectrum users, and new users (as buyers) obtain spectrum that they desperately need. This marketplace differs significantly from conventional FCC-style spectrum auctions in three aspects:

Multi-party trading with spectrum reuse. Spectrum auctions are fundamentally different from (and much more difficult than) conventional multi-unit auctions because of its unique property of reusability. Unlike the traditional goods (e.g., paintings, bonds, electricity), spectrum can be spatially reused concurrently. Although two conflicting bidders must not use the same spectrum bands simultaneously yet well-separated bidders can. While a conventional auction with \( n \) bidders and \( k \) bands can only have at most \( k \) winners, spectrum auction can have more than \( k \) winners. Therefore, unlike FCC-style auctions that have one provider (i.e. the FCC) and sell one license to only one buyer, the new marketplace supports multi-party trading. Multiple providers can selectively offer their idle spectrum pieces and each spectrum piece can be sold to multiple “small” buyers. In this way, the new marketplace can exploit spectrum reusability in spatial and temporal domains to improve spectrum usage efficiency.

On-demand spectrum trading. Instead of forcing buyers to purchase pre-defined spectrum licenses, the new marketplace enables buyer to specify their own demands. Given these demands, the marketplace intelligently selects winners and allocates spectrum to best utilize the spectrum offered by providers and support buyers. Such flexibility not only attracts a large number of participants but also enables the system to effectively multiplex spectrum supply and demand, further improving spectrum utilization.

Economic-robustness with spectrum reuse. Without good economic design, spectrum auctions can be easily manipulated by bidders, suffering huge efficiency loss. Auctioneers are forced to apply Bayesian settings, placing strong (and often wrong) assumptions on the distribution of bidder valuations [7]. The heavy overheads and the vulnerability would easily discourage both providers and players from participation. Therefore, only by preventing market manipulation can
1.3 On-demand Spectrum Auctions

Figure 1.2: A dynamic spectrum auction scenario. (Left) An auctioneer performs periodic auctions of spectrum to the bidders. (Right) A conflict graph illustrates the interference constraints among bidders.

An on-demand spectrum auction must distribute spectrum on-the-fly to a large number of bidders. Spectrum auctions are multi-unit auctions where the spectrum is divided into a number of identical channels for sale. Users wish to obtain different amount of spectrum at their desired power levels, and may be willing to pay differently depending on the assignment. Towards this goal, we need a compact bidding language to allow buyers conveniently express their desire and do it so compactly, and an efficient allocation algorithm to distribute spectrum in real-time subject to the complex interference constraints among bidders.

In this section, we discuss two ongoing efforts on spectrum allocation algorithms to support dynamic spectrum auctions. We start from a recent work [8]
that proposed a computationally-efficient auction framework with simple and effective bidding and fast auction clearing algorithms. Specifically, spectrum buyers (bidders) use a compact and yet expressive bidding format to express their desired spectrum usage and willingness to pay, while an auctioneer execute fast clearing algorithms to derive prices and allocations under different pricing models.

**Bidding Format: Piecewise Linear Price-Demand (PLPD) Bids.** Assume there are $K$ channels in total, $F_i$ is the set of channels assigned to bidder $i$, and hence the normalized spectrum assigned to $i$ is $f_i = |F_i|/K$. With PLPD, a bidder $i$ expresses the desired quantity of spectrum $f_i$ at each per-unit price $p_i$ using a continuous concave piecewise linear demand curve. That is, the bidder would like to pay $p_i \cdot f_i$ for $f_i$ channels. An PLPD curve can be expressed as a conglomeration of a set of individual linear pieces. A simple example is a linear demand curve:

$$p_i(f_i) = -a_i f_i + b_i, \quad a_i \geq 0, b_i > 0,$$

where the negative slope represents *price sensitivity* at buyers – as the per-unit price decreases, demands in general increase.

**Pricing Models.** Without considering economic-robustness, the auction pricing follows directly from each bidder’s bid. A bidder $i$ who obtains $f_i$ spectrum is charged with $p_i(f_i) \cdot f_i$ as specified by its bid. In this case, the revenue produced by each bidder is a *piecewise quadratic* function of the price:

$$R_i(p_i) = \frac{b_i p_i - p_i^2}{a_i}$$

For linear demand curves, the revenue is a quadratic function of price, with a unique maximum at $p_i = b_i/2$. We can further divide the pricing models into two types: *uniform* and *discriminatory* pricing. In uniform pricing, the auctioneer chooses a single clearing price $p$ for all the winners. Each bidder obtains a fraction of spectrum $f_i(p) = (b_i - p)/a_i$ and produces a revenue of $R_i(p) = (b_i p - p^2)/a_i$. Any bidder $i$ with $b_i \leq p$ gets no assignment. In discriminatory pricing, the auctioneer sets non-uniform clearing prices across bidders.

**Fast Auction Clearing by Linearizing the interference constraints.** Given the bids and pricing model, the auction clearing problem is to maximize the auction revenue $\sum_i R_i(p_i)$ by choosing the winners and their pricing $p_i$ subjecting to the interference constraints. This optimization problem is in general NP-hard because of the underlining interference constraints grow exponentially with the number of bidders. [8] proposed to reduce the interference constraints into a set of linearized constraints that grow linearly with the number of bidders. Specifically, it proposed the *Node-L Interference Constraints (NLI)*. Let two nodes $i$ and $j$ locate at coordinates $(x_i, y_i)$ and $(x_j, y_j)$. Node $i$ is to the *left of* node $j$ if $x_i < x_j$. If $x_i = x_j$, 

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then the node with the smaller index is considered to be to the node to the left. Then the constraint becomes: every neighbor of \( i \) to the left of \( i \), and \( i \) itself should be assigned with different channels:

\[
f_i + \sum_{j \in N_L(i)} f_j \leq 1, \quad i = 1, 2, \ldots, N
\]  \hspace{1cm} (1.3)

where \( N_L(i) \) is the set of neighbors of \( i \) lying to its left. It has been shown that the new constraints are stricter than the original constraints, and lead to a feasible but sub-optimal solution which is within a distance 3 from the optimal solution.

Using the new interference constraints, the auction clearing problem can be solved using linear programming (for uniform pricing) or separable programming \([9]\) (for discriminatory pricing). Both solutions have polynomial complexity. The readers should refer to \([8]\) for additional details on the algorithms and proofs. In practice, both algorithms run efficiently in real-time. Using a standard desktop with 3.0 GHz processor and 1 GB of RAM and assuming 3500 bidders, the auction clearing finishes in 0.05 seconds for the uniform pricing and 80 seconds for the discriminatory pricing model.

### 1.4 Economic-Robust Spectrum Auctions

When it comes to resisting market manipulation, the dominant paradigm is truthful auction design. A truthful auction guarantees that if a bidder bids the true valuation of the resource, its utility will not be less than that when it lies. Hence, the weakly-dominating strategy for a bidder is to bid its true valuation. As we will show, a truthful auction charges a winner independent of its actual bid, which is different from the auction design in the previous section. To bidders, a truthful auction eliminates the expensive overhead of strategizing about other bidders and prevents market manipulation. Thus it can attract a wide range of network nodes/establishments to engage in the marketplace. To the auctioneer, by encouraging bidders to reveal their true valuations, a truthful auction can help the auctioneer increase its revenue by assigning spectrum to the bidders who value it the most. For the same reason, many classical auction systems are made truthful, including the sealed-bid secondary-price \([10]\), k-position \([11, 12]\), and VCG auctions \([13, 14]\).

While prior works have enforced truthfulness in conventional auctions, existing truthful designs either fail or become computationally prohibitive when applied to spectrum auctions. The fundamental reason is that unlike goods (e.g. paintings, bonds, electricity) in conventional auctions, spectrum is reusable among bidders subjecting to the spatial interference constraints. Because interference is only a local effect, bidders in close proximity cannot use the same spectrum frequency simultaneously but well-separated bidders can. These heterogeneous inter-dependencies
among bidders make secondary-price and k-position auctions no longer truthful. Furthermore, these constraints make the problem of finding the optimal spectrum allocation NP-complete, and hence a real-time spectrum auction with many bidders must resort to greedy allocations that are computationally efficient. Unfortunately, it has been shown that the VCG auction loses its truthfulness under greedy allocations.

In the following, we describe VERITAS [5], a truthful dynamic spectrum auction framework. VERITAS achieves truthfulness with computationally-efficient spectrum allocation and pricing mechanisms, making it feasible for the online short-term auction. In addition, VERITAS provides the auctioneer with the capability and flexibility of maximizing its customized objective, and allows bidders to request spectrum by the exactly number of channels it would like to obtain, or by a range defined by the minimal and maximal number of channels.

Consider a typical sealed-bid auction in Figure 1.2. The auctioneer sells \( k \) channels by running an online auction periodically. Each bidder requests spectrum by the number of channels and the per-channel price it would like to pay. After receiving the bids, the auctioneer determines the winners, their spectrum allocations and prices, based on the bids and the interference condition among bidders. As shown in Figure 1.2, the interference condition is represented by a conflict graph \( G = (V, E) \), where \( V \) is the collection of the bidders and \( E \) is the collection of edges where two bidders share an edge if they conflict. Table 1.1 summarizes the notations used to define an auction problem.

Using these notations, we now define a truthful auction, and a truthful and efficient spectrum auction:

**Definition 1** A truthful auction is one in which no bidder \( i \) can obtain higher utility \( u_i \) by setting \( b_i \neq v_i \).

In the context of spectrum auctions, the design must ensure truthfulness and enable spectrum reuse across auction winners to improve spectrum utilization.

**Definition 2** An efficient and a truthful spectrum auction is one which is truthful and maximizes the efficiency of spectrum usage subject to the interference constraints.

In building a truthful and efficient spectrum auction, VERITAS integrates a greedy spectrum allocation with a carefully designed pricing mechanism. Let’s start from a simple scenario where bidders’ channel requests are strict: a bidder \( i \) requests \( d_i \) channels and only accepts allocations of either 0 or \( d_i \) channels.

**Spectrum Allocation.** In determining the auction winners, VERITAS applies a greedy solution. It first sorts the bid set \( B \) by a descending order of \( b_i \), and then allocates bidders sequentially from the highest one to the lowest one. Form each
1.4 Economic-Robust Spectrum Auctions

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel request $d_i$</td>
<td>The number of channels requested by bidder $i$</td>
</tr>
<tr>
<td>$D = {d_1, d_2, \ldots, d_n}$</td>
<td>The set of demands across all the bidders.</td>
</tr>
<tr>
<td>Per-channel bid $b_i$</td>
<td>The per-channel bid submitted by bidder $i$, or the maximum price bidder $i$ is willing to pay for a channel</td>
</tr>
<tr>
<td>$B = {b_1, b_2, \ldots, b_n}$</td>
<td>The set of bids submitted by all the bidders.</td>
</tr>
<tr>
<td>Per-channel valuation $v_i$</td>
<td>The true valuation a bidder $i$ has for a channel. In most cases, $v_i$ is private and known only to bidder $i$.</td>
</tr>
<tr>
<td>Channel allocation $d_a^i$</td>
<td>The number of channels an auction winner $i$ receives</td>
</tr>
<tr>
<td>Clearing price $p_i$</td>
<td>The price charged to an auction winner $i$. In a truthful auction, $p_i \leq d_a^i \cdot v_i$.</td>
</tr>
<tr>
<td>Bidder utility $u_i$</td>
<td>The utility of bidder $i$, or the residual worth of the channels. $u_i = v_i \cdot d_a^i - p_i$ if $i$ is an auction winner and 0 otherwise.</td>
</tr>
</tbody>
</table>

Table 1.1: Summary of auction notations.

bidder $i$, the algorithm first checks whether there are enough channels to satisfy $i$'s request $d_i$. If so, it assigns $i$ with $d_i$ lowest indexed channels that have not been assigned to $i$'s conflicting peers. Such monotonic allocation is critical to achieve auction truthfulness.

**Winner Pricing.** VERITAS charges each winner $i$ with the bid of its critical neighbor multiplied by the number of channels allocated to $i$. The price reflects the minimum value of $i$'s bid to win the auction, and is independent of $i$'s actual bid, and is always no more than $i$'s actual bid multiplied by the number of channels allocated to $i$. This property is also referred to as “individual rationality.” The critical neighbor is defined as follows:

**Definition 3** Given $\{B \setminus b_i\}$, a critical neighbor $C(i)$ of bidder $i$ is one of $i$’s neighbors where if $i$ bids lower than $C(i)$, $i$ will not be allocated, and if $i$ bids higher than $C(i)$, $i$ will be allocated.

At the first sight, finding the critical neighbor for each bidder $i$ seems computationally expensive. It requires inserting $i$’s bid immediately after each of its neighbors and running allocation algorithm repeatedly. VERITAS overcomes this problem using an intelligent pricing algorithm that identifies the critical neighbor for each bidder by running the allocation algorithm once. For each bidder $i$, the algorithm first removes $i$ from the sorted bid set and runs the allocation. When assigning channels to $i$’s conflicting peers, the algorithm removes the assigned channels from $i$’s available channel set. The first winning conflicting peer who makes $i$’s available
channels less than its demand $d_i$ is $i$’s critical neighbor. The detailed algorithm description can be found in [5].

**Supporting Other Bidding Formats.** VERITAS enables bidders to use diverse demand formats. A bidder can request spectrum by the exactly number of channels it would like to obtain (strict requests), or by a range defined by the minimal and maximal number of channels (range requests). Using the range request, a bidder $i$ can request $d_i$ channels but accept any number of channels between 0 and $d_i$. To ensure truthfulness under this request, VERITAS applies an advanced allocation and pricing mechanism. When allocating channels, if the number of available channels is less than $i$’s demand $d_i$, the algorithm allocates whatever is possible. When determining prices, the algorithm needs to find multiple (rather than one) critical neighbors for each winner because bidding below each critical neighbor will result into the allocation of different number of channels. For each set of additional channels obtained by bidding higher than the last critical neighbor, the algorithm charges the winner with the bid of its last critical neighbor. The clearing price is the sum of prices charged for all of the bidder’s assigned channels.

**Supporting Different Auction Objectives.** VERITAS provides the auctioneer with the capability and flexibility of maximizing its customized objective. By sorting the bid set differently, the auctioneer can configure the order of allocation to maximize the auction revenue or the social welfare. For example, it has been shown that to maximize the sum of winning bids, known as the social welfare [12], the best-known greedy algorithm is to assign channel following the descending order of $\frac{b_i}{|N(i)+1|}$ [16] where $N(i)$ is the number of conflict peers of bidder $i$. VERITAS enables this flexibility by allowing different sorting metrics as long as it is an increasing function of the bid $b_i$, and not affected by the bids of other bidders, such as $b_i / (|N(i)|+1)$ or $b_i \cdot |N(i)|$.

**VERITAS Performance and Complexity.** It has been shown that the VERITAS auction design is truthful by combining the monotonic spectrum allocation and the critical-neighbor based pricing algorithm [5]. The computational complexity of VERITAS is in the order of $O(N^3K)$ where $N$ is the number of bidders, and $K$ is the number of channels auctioned. Among them, $O(N \log N + K|E|)$ is from the allocation algorithm and $O(NK|E|)$ is from the pricing algorithm), where $|E|$ is the number of edges in the bidder conflict graph. Such polynomial complexity makes VERITAS suitable for dynamic, on-demand spectrum auctions.

Figure 1.3(a) compares VERITAS’ spectrum utilization to that of the best-known greedy allocation algorithm [16], where VERITAS performs similarly to the greedy solution. Figure 1.3(a) examines its auction revenue as a function of the number of channels auctioned. VERITAS exhibits an interesting trend: as the number of channels auctioned increases, the revenue first increases and then
Figure 1.3: VERITAS performance. (a) Spectrum allocation efficiency vs. the number of bidders. VERITAS performs similarly to the best known greedy algorithm [16]. (b) VERITAS auction revenue vs. the number of channels auctioned. The auction revenue depends heavily on the level of bidder competitions. As the number of channels auctioned increases, the level of competition decreases and the winners’ prices reduce.

1.5 Double Spectrum Auctions for Multi-party Trading

We have described an auction design where the auctioneer sells its spectrum channels to buyers. In this section, we describe a double spectrum auction design where multiple spectrum sellers and buyers can trade spectrum flexibly by interacting with an auctioneer. As shown in Figure 1.4, the auctioneer is a match-maker between sellers and buyers. It buys spectrum pieces from the sellers and sell them to the buyers. In this way, existing spectrum owners (as sellers) can obtain financial gains by leasing their selected idle spectrum to new spectrum users; new users (as buyers) can access the spectrum they desperately need and in the format they truly desire. By multiplexing spectrum supply and demand in time and space, dynamic
Figure 1.4: Multi-party spectrum trading based on double auctions. The auctioneer performs an auction among both sellers and buyers. Sellers provide idle spectrum pieces dynamically with regional coverage, while buyers request spectrum channels in local areas based on their demands. Each channel contributed by a seller can be reused by multiple non-conflicting buyers.

Auctions can significantly improve spectrum utilization.

To model a double spectrum auction, we define the bid, true valuation, price and utility of both sellers and buyers. The notations for buyers, \( B^b_n, V^b_n, P^b_n \) and \( U^b_n \) follow those in Table 1.1, and the notations for sellers are defined in Table 1.2. In addition to truthfulness and spectrum reuse, a double spectrum auction must also achieve two additional properties: individual rationality and budget balance.

**Definition 4** A double auction is **individual rational** if no winning buyer pays more than its bid (i.e. \( P^b_n \leq B^b_n \)), and no winning seller gets paid less than its bid (i.e. \( P^s_m \geq B^s_m \)).

This property guarantees non-negative utilities for bidders who bid truthfully, providing them the incentives to participate.

**Definition 5** A double auction is **ex-post budget balanced** if the auctioneer’s profit \( \Phi \geq 0 \). The profit is defined as the difference between the revenue collected from buyers and the expense paid to sellers: \( \Phi = \sum_{n=1}^{N} P^b_n - \sum_{m=1}^{M} P^s_m \geq 0 \).

This property ensures that the auctioneer has incentives to set up the auction.

In the following, we describe TRUST [6], a new double spectrum auction framework that achieves the four required properties: spectrum reuse, truthfulness, individual rationality and budget balance. Table 1.3 compares TRUST to existing double auction designs. Conventional double auction designs (VCG [17] and McAfee [18]) achieve truthfulness but do not consider spectrum reusability. VERITAS [5] only addresses single-sided buyer-only auctions, and loses the truthfulness when directly extended to double auctions [6].
1.5 Double Spectrum Auctions for Multi-party Trading

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller’s per-channel bid $B_m^s$</td>
<td>The minimum payment required by seller $m$ to sell one channel.</td>
</tr>
<tr>
<td>Seller’s per-channel valuation $V_m^s$</td>
<td>The true valuation a seller $m$ has for a channel.</td>
</tr>
<tr>
<td>Seller’s price $P_m^s$</td>
<td>The payment a winning seller $m$ receives by selling a channel.</td>
</tr>
<tr>
<td>Seller’s utility $U_m^s$</td>
<td>The utility of seller $m$ $U_m^s = P_m^s - V_m^s$ if $m$ wins the auction and 0 otherwise. This is different from the buyer case.</td>
</tr>
</tbody>
</table>

Table 1.2: Summary of double auction notations related to sellers.

<table>
<thead>
<tr>
<th>Existing Double Auction Designs</th>
<th>Spectrum Reuse</th>
<th>Truthfulness</th>
<th>Ex-post Budget Balance</th>
<th>Individual Rationality</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCG</td>
<td>X</td>
<td>√</td>
<td>X</td>
<td>√</td>
</tr>
<tr>
<td>McAfee</td>
<td>X</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>VERITAS extension</td>
<td>√</td>
<td>X</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>TRUST</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

Table 1.3: Comparison of various double auction designs.

TRUST [6] breaks the barrier between spectrum reuse and economic-robustness in double spectrum auctions. In essence, it enables spectrum reuse by applying a spectrum allocation algorithm to form buyer groups. It achieves the three economic properties via the bid-independent group formation and a reusability-aware pricing mechanism. TRUST consists of three components:

**Grouping Buyers** TRUST groups multiple non-conflicting buyers into groups so that buyers in each group do not conflict and can reuse the same channel. This is done privately by the auctioneer performing a spectrum allocation algorithm and grouping buyers assigned to the same channel to a group. Unlike VERITAS, the group formation is independent of the buyer bids to prevent bidders from rigging their bids to manipulate its group size and members.

The group formation can cope with various interference models by using different spectrum allocation algorithms. If the buyer interference condition is modeled by a conflict graph, the group formation is equivalent to finding the independent sets of the conflict graph [19, 20]. If the condition is modeled by the physical Signal
to Interference and Noise Ratio (SINR) [21], TRUST finds multiple sets of buyers who can transmit simultaneously and maintain an adequate received SINR [22]. Assuming channels are homogeneous, TRUST performs this allocation only to form buyer groups, not to assign specific channels to buyers.

**Determining Winners.** Next, TRUST treats each buyer group as a super-buyer and runs the conventional double spectrum auction algorithm to determine the winning sellers and super-buyers. Let $G_1, G_2, ..., G_L$ represent the $L$ groups formed. For any group $G_l$ with $n_l = |G_l|$ buyers, the group bid $\pi_l$ is:

$$\pi_l = \min\{B^b_n|n \in G_l\} \cdot n_l.$$  \hspace{1cm} (1.4)

TRUST sorts the seller bids in non-decreasing order and the buyer group bids in non-increasing order: $B^s_1 \leq B^s_2 \leq \ldots \leq B^s_M$, and $\mathbb{B}^g : \pi_1 \geq \pi_2 \geq \ldots \geq \pi_L$. Define $k$ as the last profitable trade:

$$k = \arg\max_{l \leq \min\{L,M\}} \pi_l \geq B^s_l.$$  \hspace{1cm} (1.5)

Then the auction winners are the first $(k-1)$ sellers, and the first $(k-1)$ buyer groups.

**Pricing.** To ensure truthfulness, TRUST pays each winning seller $m$ by the $k$th seller’s bid $B^s_k$, and charges each winning buyer group $l$ by the $k$th buyer group’s bid $\pi_k$. This group price is evenly shared among the buyers in the group $l$:

$$P^b_n = \pi_k/n_l, \quad \forall n \in G_l.$$  \hspace{1cm} (1.6)

No charges or payments are made to losing buyers and sellers. The uniform pricing is fair because buyers in a winning group obtain the same channel, thus should be charged equally. In addition, to ensure individual rationality, a group bid must not exceed the product of the lowest buyer bid in the group and the number of buyers in the group, which is used in the process of determining winning groups. With such pricing mechanism, the auctioneer’s profit becomes $\Phi = (k-1) \cdot (\pi_k - B^s_k)$ and and it is easy to show that $\Phi \geq 0$.

**TRUST Performance and Complexity.** As shown in [6], by integrating the monotonic winner determination and the bid-independent pricing, TRUST achieves truthfulness, ex-post budget balance, and individual rationality while enabling spectrum reuse to improve spectrum utilization. One key advantage of TRUST is that it can use any spectrum allocation algorithm in forming buyer groups. Thus its complexity depends heavily on the allocation algorithm used.

On the other hand, ensuring these economic properties comes at a cost in spectrum utilization. This is because TRUST selects winning buyer groups by the minimum bid in the group multiplied by the group size, so that groups of different
sizes have equal opportunity in being chosen. On the other hand, the convectional spectrum allocation algorithms always choose large groups, leading to an advantage in spectrum utilization. Figure 1.5 illustrates the ratio of TRUST’s spectrum utilization over that of conventional spectrum allocations without economic consideration [19, 20]. It includes TRUST with four spectrum allocation algorithms, and examines the performance using random and clustered topologies. In random network topologies, TRUST achieves 70–80% spectrum utilization of the conventional spectrum allocation, while in clustered topologies, TRUST sacrifices roughly 50% of spectrum utilization in exchange for economic robustness. This is because in clustered topologies, the group sizes become much more diverse, and TRUST could select a set of small buyer groups which degrades the overall spectrum utilization.

1.6 Further Reading

Exploitation of market mechanism for dynamic allocation and redistribution of spectrum in cognitive radio networks has been the topic of several other recent research investigations, and the literature on this topic is growing. Importantly also, the use of such mechanism are starting to move from the realm of pure research into that of development and commercial exploitation. For example, Spectrum Bridge Inc. (SBI) [23], a US-based company has developed a real time online market place that enables spectrum owners and users to buy, sell and lease FCC licensed spectrum. According to the company’s web site its online market place, SpecEx, provides access to over 200 billion of spectrum that the FCC has made eligible for
secondary market transactions. For the benefit of reader we shall summarise in the remaining this section some of the most recent research on the use of market mechanism for dynamic spectrum access.

A framework for coordinating dynamic spectrum access aiming service providers was proposed in [3]. The scheme proposed in this work relies on a spectrum broker that controls the allocation of spectrum among the spectrum requesting operators. This work was later extended to cases where the interference among bidders is modeled by pairwise and physical interference models and the bidders can bid for heterogeneous channels of different width using generic bidding functions [24].

The price dynamics of a dynamic spectrum market was explored in [25]. The authors considered a market place consisting of spectrum agile network service providers and users. Competition among multiple primary users to sell their spectrum are modeled in this work as a non-cooperative game. An interesting feature of this work is that the analysis takes into account differences in evaluation of the quality of the offered spectrum by buyers. For example, spectrum at lower frequencies, such as UHF, travel longer distances and penetrate more readily through walls. Therefore such bands may value such spectrum bands highly for applications that require good penetration properties. Also, depending on their operating wireless technology some spectrum buyers may value contiguous segments of spectrum higher than non-contiguous ones.

The dynamics of multiple-seller and multiple-buyer spectrum trading in dynamic spectrum access networks is also considered in [26]. In this work it is assumed that the secondary users can adapt their spectrum buying behavior to the variations in price and quality of spectrum offered by different primary users. At the same time the primary users can adjust their behavior in selling their spectrum in order to achieve the highest utility. Similar to [25] the competition among primary users in selling spectrum is modelled using a non-cooperative game formulation. At the same time, evolution in the spectrum buying behavior of secondary users are analyzed by using the deterministic and stochastic models of evolutionary games.

One of the early papers that explores the use of auctions for dynamic allocation of spectrum is [27]. The authors consider a scenario where multiple code division multiple access (CDMA) operators bid for the spectrum to a spectrum manager. They present an optimal bidding and pricing mechanism was presented with the objective of maximising the revenue of the operators based on the willingness of users' to pay. Auction-based mechanism for dynamic spectrum access are also explored in [28], where and optimisation problem is formulated to maximize the revenue of spectrum owners through pricing and spectrum assignment. In [4] the authors describe a combinatorial clock auction mechanism for trading of spectrum in the context of and OFDMA-based cognitive radio network. Combinatorial clock auctions [29] are used when there are a range of items on sale which may be logically grouped together into many different packages to suit either the buyer, the seller
or both. In these auctions bids for such packages are made throughout a number of sequential open rounds and then a final sealed-bid round. During the sequential rounds buyers have an opportunity to explore the bid-space as their bids are either accepted or rejected until there is no change in the winners or no new bids are submitted. The authors of [4] present a modified version of the combinatorial clock auctions in order to reduce the complexity of the mechanism for cognitive radios that attempt to buy and sell spectrum on behalf of users.

1.7 Chapter Summary

In this chapter we have examined the challenges and solutions in the area of spectrum trading. Different from the conventional Command & Control management model, spectrum trading is an open, market-based approach for redistributing the spectrum where new users can gain access to the spectrum they desperately need and existing owners can gain financial incentives to “lease” their idle spectrum. We have focused mainly on dynamic spectrum auctions because auctions are among the best-known market-based allocation mechanisms. Dynamic spectrum auctions are fundamentally different from (and much more difficult than) conventional multi-unit auctions because of their unique requirement of spectrum reusability. With this in mind, we have introduced three recent works on on-demand spectrum auctions, truthful spectrum auctions and truthful double spectrum auctions. Together, they provide the basic building blocks for constructing an efficient, economic-robust and real-time dynamic spectrum marketplace.

It is important to note that there have been numerous contributions and ongoing efforts on dynamic spectrum allocation, pricing, trading and auctions. A small set of them were summarized in the Further Reading section. Building on these extensive contributions, the use of spectrum trading is moving from pure research to several commercial deployments, and hopefully will expand to the general public in near future.

1.8 Chapter Questions

1. What is spectrum trading? How does it differ from the Command and Control Model?

2. This chapter deals mainly with auction-based mechanisms to enable efficient deals between spectrum buyers and sellers. Name and explain at least one other market mechanism that can be used for trading spectrum.

3. How does spectrum differ from other natural resources such as gas and electricity? How do these differences impact the use of auctions in trading spec-
4. Explain what a conflict graph is. Use MATLAB to construct and visualize conflict graphs for a collection of 100 nodes uniformly distributed in a 1 km$^2$ rectangular area. Assume that all nodes have a 100 m transmission radius.

5. What are the limitations of FCC-style auctions?

6. What are the differences between single-sided auctions and double auctions?

7. Consider the first-price auction where the winner is charged by its bid, what is the revenue trend would you expect as the number of channels auctioned increases? Explain your conjecture by comparing it to Figure 1.3.

8. Critique neighbour is defined in VERITAS for determining each winner’s price. Consider a winner $i$ in VERITAS, and among $i$’s unallocated neighbors (i.e. $i$’s neighbors who did not win any channel in the auction), let $j$ be the one with the highest bid. Is $j$ always $i$’s critique neighbour? If it is, explain the reason, and if it is not, give a counter example.

9. Buyer group formation is an important step for TRUST to enable spectrum reuse. Given the conflict graph of buyers and the set of bids of sellers, do you think it is a good idea to make the buyer group size more balanced? Explain your conclusion.
Bibliography


BIBLIOGRAPHY


