Nonlinear Distortion in SPAD-Based Optical OFDM Systems

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Abstract—In this paper, an optical orthogonal frequency division multiplexing (OFDM) system based on a single-photon avalanche diode (SPAD) receiver with a nonlinear distortion is presented. In this study, the methods to calculate the output of passive and active quenching SPAD arrays are given. Nonlinear performances of these two SPAD arrays and their effects on bit-error ratio performances of SPAD-based DC-biased optical OFDM and asymmetrically clipped optical OFDM are simulated in the absence of background light. In the simulation, the maximum optical irradiance and the low error area are given as the metrics of the nonlinear distortion. The effects of the nonlinear distortion are discussed for different modulation schemes, symbol rates and spectral efficiencies.

Index Terms—optical wireless communication (OWC), single-photon avalanche diode (SPAD), nonlinear distortion, optical OFDM.

I. INTRODUCTION

Currently high speed light emitting diodes (LEDs) and laser diodes (LDs) are used as transmitters in optical wireless communication (OWC) systems. With a single LED, an OWC system can achieve speeds exceeding 3 Gb/s [1]. However, the incoherent light output of the transmitters means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. Thus, OWC systems are usually considered to be modulated as an intensity modulation (IM) and direct detection (DD) system [2]. Unipolar modulation schemes such as on-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) can be used in conjunction with IM/DD systems [3]. In order to fully use the limited modulation bandwidth of the device and achieve high data rates, orthogonal frequency division multiplexing (OFDM) is applied in OWC systems by utilizing adaptive bit and power loading [1]. Unlike OFDM in radio frequency, optical OFDM (O-OFDM) requires real valued signals which are generated by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. However, this decreases the spectral efficiency by half. Diverse O-OFDM modulation schemes have been realized and utilized in OWC, such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), unipolar OFDM (U-OFDM) and non-DC-biased OFDM (NDC-OFDM) [4], [5].

Typically, highly sensitive photodiodes (PDs) such as positive-intrinsic-negative (PIN) diodes and avalanche photodiodes (APDs) are applied as receivers in OWC. However, when the OWC system is applied in low optical power and long distance transmission, such as in a gas well downhole monitoring system [6] and data transmission over plastic optical fibres [7], the number of photons reaching the receivers are significantly less than in standard indoor OWC links. In these scenarios, conventional PDs have unsatisfactory performance because the transimpedance amplifier (TIA) significantly reduces the sensitivity of the receiver and limits the signal-to-noise ratio (SNR). As a consequence, those low power signals are buried in noise. Hence, when compared with conventional PDs, single-photon avalanche diodes (SPADs) would be more suitable receivers in those scenarios. The SPAD detector does not require a TIA and thus the output signal is not distorted by thermal noise. In addition, as SPADs can detect even a single photon, a bit of information-carried photons can be received accurately. Therefore, the SPAD receiver can perform at significantly higher sensitivity and optical power efficiency than conventional PDs. In a previous work [8], an O-OFDM system with a SPAD receiver was presented and compared with state-of-the-art PD-received based O-OFDM systems. When the transmission speed is 1 Mbps, SPAD-based OFDM enhances the sensitivity by a staggering 30.5 dB over the PD-based system.

However, a SPAD receiver can only detect one photon within a device specific dead time which constrains the ability to recover a signal. In addition, since the output of the detector is a photon count value, there is a maximum number of photons that the system can detect. This limits the maximum tolerable optical irradiance which results in a receiver nonlinear distortion. This study investigates the performance of the SPAD-based OFDM system when the nonlinear distortion of the SPAD receiver occurs.

The rest of this paper is organized as follows. The system model of the SPAD-based OFDM system is described in Section II. The nonlinear distortion of the SPAD receiver is presented in Section III. The simulation results of the system and discussion on the nonlinear distortion of the system are given in Section IV. Conclusions are given in Section V.
II. SYSTEM MODEL

Fig. 1 illustrates the system model of OFDM with SPAD receivers.

A. Optical OFDM Modulation

At the transmitter, the input bit stream is transformed into complex symbols, $X(n)$, by a $M$-quadrature amplitude modulation (QAM) modulator, where $M$ is the constellation size. The symbols are allocated to $N$ subcarriers, $X(k)$, $k = 0, \cdots, N-1$. In OFDM, $N$ denotes the size of IFFT/FFT, where $N$ is set to 2048. In general, two standard techniques, DCO-OFDM and ACO-OFDM, are used to obtain positive and real-valued OFDM symbols [9]. In DCO-OFDM, $N/2-1$ symbols in $X(n)$, $n = 1, \cdots, N/2-1$, are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. In ACO-OFDM, $N/4$ QAM symbols in $X(n)$, $n = 1, \cdots, N/4$, are mapped on to half of the odd subcarriers of the OFDM frame, $X(k)$, $k = 1, 3, 5, \cdots, N/2-1$. At the same time, the even subcarriers are set to zero. In both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols through the IFFT block. Since transmitters can only send unipolar signals, the real-valued OFDM symbols need to be clipped. In DCO-OFDM, a DC bias is added to make the signal unipolar [9]. In practice, the value of the DC bias, which is related to the average power of the OFDM symbols, is defined as:

$$B_{DC} = \alpha \sqrt{E\{x^2(k)\}},$$

(1)

where $x(k)$ is the OFDM symbol frame vector; and $10 \log_{10}(\alpha^2 + 1)$ is defined as the bias level in dB. The bias level in the current simulations is set to 7 dB and 13 dB, which are adopted from [8] for consistency. After the DC-bias, the OFDM frame is simply clipped by:

$$x_{\text{clipped}}(k) = \begin{cases} x_{\text{biased}}(k), & x_{\text{biased}}(k) \geq 0, \\ 0, & x_{\text{biased}}(k) < 0, \end{cases}$$

(2)

where $x_{\text{biased}}(k)$ is the DC-biased symbol which is calculated as $x_{\text{biased}}(k) = x(k) + B_{DC}$. The clipped unipolar symbol is denoted by $x_{\text{clipped}}(k)$. In ACO-OFDM, since symbols are antisymmetric, clipped unipolar symbols are obtained by setting the negative part to zero. In the simulation, after being transformed into an optical intensity signal, the clipped signal is transmitted by an LED transmitter.

B. SPAD Receiver

A SPAD is an APD which is biased beyond reverse breakdown in the so-called ‘Geiger’ region. In this mode of operation, a SPAD triggers billions of electron-hole pair generations for each detected photon. In other words, in ‘Geiger’ mode, a SPAD generates a very large current by receiving a single photon and thus can essentially be modelled as a single photon counter. The photodetection process of an ideal photon counter can be modelled using Poisson statistics which describe the shot noise effect ( [10] and references therein).

In this study, in order to increase the capacity of the photon counts, an array of SPADs which outputs the superposition
of the photon counts from the individual SPADs is considered [11]. Five important parameters of the SPAD array are introduced as follows.

1) **Fill Factor (FF):** FF is the ratio of the total SPAD active area to the total array area. For the SPAD array, FF represents the probability that a photon hits the active area. If the photon triggers an avalanche, it will be counted. In other words, the percentage of photons in a beam of light reaching the active area can be approximated to FF. In this study, the value of FF is denoted by $C_{FF}$.

2) **Photon Detection Probability (PDP):** PDP is the probability that a photon hitting the active area triggers an avalanche. This avalanche will generate a pulse which can be counted by an accumulator. The accumulator will give the output of the array. PDP is different to the quantum efficiency of a conventional PD, in which the quantum efficiency sometimes includes fill factor effects [12]. In this study, the value of PDP is denoted by $C_{PDP}$.

3) **Dark Count Rate (DCR):** A thermally-generated carrier can also trigger an avalanche which increases the array output. Even in complete darkness, this phenomenon still exists as long as the SPAD devices are opened. The average number of counts in darkness per second is referred to as DCR which is regarded as a fixed signal-unrelated noise of SPAD. In this study, the average DCR of a single SPAD device is denoted by $N_{DCR}$.

4) **After Pulsing Probability (APP):** After pulses are correlated to detections by the time dependent release of trapped carriers [11]. Additional avalanches are triggered after receiving a photon or a dark photon. This means that the after pulsing effect will also increase the array output related to both the incoming signal and the dark counts. The delayed counts will bring inter-symbol interference due to the high data rate. But in low speed transmission, as the sample period is much longer than the delayed time, the after pulsing effect has a negligible effect on the next sample period. In this study, the value of APP is denoted by $P_{AP}$.

5) **Dead Time:** After an avalanche is triggered, whether caused by the signal photons or dark photons, the SPAD device needs to be actively or passively recharged in a short period of time and we refer this short period of time as dead time. During this time, the SPAD device is unable to detect further signal photons or dark photons. In other words, each individual SPAD in the array can only receive one photon during the dead time. In this study, the value of the dead time is denoted by $T_d$.

Fig. 2 illustrates the system model of the SPAD array receiving optical signals. In order to generate received O-OFDM samples, the output of the SPAD array is counted over a short-time average period, $T_{ST}$, at time instances $t_k = kT_s$ of the received optical signal $x_r(t)$. Note that $T_s$, which denotes the sampling period of the time domain OFDM signal at the transmitter, is assumed to be longer than $T_{ST}$. These photon counts are denoted by $\nu(k)$ which is the superposition of the photon counts from each individual SPADs, $a_m(k)$, as shown in Fig. 2:

$$\nu(k) = \sum_{m=1}^{N_{SPAD}} a_m(k),$$

where $N_{SPAD}$ is the number of SPAD devices in the array. As the photon counts from each individual SPAD can be approximately modelled using Poisson statistics, the photon counts at the output of the SPAD array (i.e., $\nu(k)$) can be still described by a Poisson distribution:

$$Pr(\nu(k) = j) = \exp(-\mu(k)) \frac{\mu(k)^j}{j!},$$

where the average photon counts $\mu(k)$ can be expressed as a function of the received signal and the effects of the SPADs’ parameters:

$$\mu(k) = \left[ C_{FF} C_{PDP} \int_{t_{s}}^{t_{s}+T_{ST}} x_r(t) dt + n_{DCR} \right] (1 + P_{AP}),$$

where $E_P$ denotes the energy of a photon which is calculated by $\frac{hc}{\lambda_L}$. Planck’s constant is represented by $h$; $c_L$ is the speed of the light; and $\lambda_L$ is the light wavelength of the LED transmitter. The noise caused by dark counts is denoted by $n_{DCR} = N_{DCR} N_{SPAD} T_{ST}$.

### C. Optical OFDM Demodulation

The output of the SPAD array is the number of photons, $\nu(k)$, and the system is designed based on a conventional O-OFDM demodulator which requires the amplitude of the electrical signal (optical power) to demodulate the received signal to the original encoded bits. Thus, the photon-to-amplitude equalizer is used to simply convert the received photon number to the corresponding electrical signal amplitude (optical power), $x'(k)$. The coefficient of the equalizer is calculated by a pilot which can record the effect of the attenuation.

Assuming that there is no other distortion effects during the transmission, the recovered signal, $x'(k)$, can be scaled to the original clipped signal, $x_{clipped}(k)$. The recovered OFDM symbols from the SPAD are passed through a FFT operation which converts symbols to the frequency domain. In DCO-OFDM, $N/2 - 1$ symbols are obtained from the corresponding subcarriers to constitute a QAM symbol frame, $X'(n)$. In ACO-OFDM, $N/4$ symbols are obtained. The detected QAM symbols are then decoded by the conventional maximum likelihood (ML) estimator in order to obtain the output bit stream.

### III. NONLINEAR DISTORTION IN SPAD

The complete model of nonlinear distortion in O-OFDM systems has been presented in [13]. In the SPAD-based system, a special form of nonlinear distortion which is caused by the saturation of SPAD devices should be taken into consideration. In this study, passive quenching (PQ) and active quenching (AQ) SPADs are considered and compared.
A. PQ SPAD

The configuration of the PQ circuit is presented in [14]. PQ SPADs are identified as paralyzable detectors where any counts occurring during the dead time (including signal, dark count and after pulse) are not registered, but are extending the dead time. For a single device, the relationship between the average incoming photon counts per second, $\mu_m$, and the measured one, $\mu_{PQm}$, is [14]:

$$\mu_{PQm} = \mu_m \exp(-\mu_m \tau_d). \quad (6)$$

Thus, for each $T_{ST}$, the average number of the measured photon counts, $\mu_{PQm}(k)$, is calculated by:

$$\mu_{PQm}(k) = \frac{\mu_m(k)}{T_{ST}} \exp \left( -\frac{\mu_m(k) \tau_d}{T_{ST}} \right) T_{ST}$$

$$= \mu_m(k) \exp \left( -\frac{\mu_m(k) \tau_d}{T_{ST}} \right). \quad (7)$$

where $\mu_m(k)$ denotes the average actual incoming photon counts in the same $T_{ST}$. For the SPAD array, $\mu_m(k)$ is equal to $\mu(k)/N_{SPAD}$. In this study, if the SPAD array is composed by PQ SPAD devices, the average output of the array during each $T_{ST}$ can be expressed as:

$$\mu_{PQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{PQm}(k)$$

$$= \mu(k) \exp \left( -\frac{\mu(k) \tau_d}{T_{ST}N_{SPAD}} \right). \quad (8)$$

B. AQ SPAD

Compared with PQ SPAD, the configuration of AQ SPAD is more complex and requires more area and power, but when any event arrives during the dead time, the additional events are not registered and do not prolong the dead time. Thus AQ SPADs are defined as non-paralyzable detectors and have higher count rates than PQ SPADs. For a single AQ SPAD, the average measured photon counts per second, $\mu_{AQm}$, can be expressed as a function of $\mu_m$ [14]:

$$\mu_{AQm} = \frac{\mu_m}{1 + \mu_m \tau_d}. \quad (9)$$

For each $T_{ST}$, the average output of a single device is:

$$\mu_{AQm}(k) = \frac{\mu_m(k)}{1 + \mu_m(k) \tau_d}. \quad (10)$$

Thus the average output of the AQ SPAD array in $T_{ST}$ is:

$$\mu_{AQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{AQm}(k)$$

$$= \frac{\mu(k)}{1 + \frac{\mu(k) \tau_d}{T_{ST}N_{SPAD}}}. \quad (11)$$

C. Nonlinear Distortion Problem

From (8) and (11), it is apparent that the real average outputs of the SPAD arrays have a nonlinear relationship with the average incoming number of photons. When the incoming photon rate is low, the outputs of each array are nearly equal to the inputs. However, when the SPAD devices are nearly saturated, a nonlinear distortion appears. For the PQ SPAD array, after reaching the maximum photon count rate, $\mu_{PQ_{max}}$, as the PQ SPAD devices are paralyzed, the outputs of the SPAD array rapidly decreases with an increasing rate of incoming photons. From (8), $\mu_{PQ_{max}}$ can be calculated:

$$\mu_{PQ_{max}} = \frac{T_{ST}N_{SPAD}}{\exp(1)\tau_d}. \quad (12)$$

For AQ SPAD, when the incoming photon rate increases, the devices are non-paralyzed but the outputs dramatically converge to the maximum photon rate, $\mu_{AQ_{max}}$. In other words, if the potential photon counts, including signal photons, dark photons and after pulse events, are more than $\mu_{AQ_{max}}$, the AQ SPAD array will be saturated. The photon counts at the output of the SPAD array are constrained to $\mu_{AQ_{max}}$ and the extra photons are refused and lost. From (11), $\mu_{AQ_{max}}$ can be obtained:

$$\mu_{AQ_{max}} = \frac{T_{ST}N_{SPAD}}{\tau_d}. \quad (13)$$

In the SPAD-based Optical OFDM system, some high amplitude symbols in the recovered signal ($\chi'(k)$) are distorted by PQ and AQ recharged circuits resulting in loss of information.

IV. RESULTS AND DISCUSSION

In this study, the performance of SPAD-based DCO-OFDM and ACO-OFDM with the nonlinear distortion are simulated and compared. The optical signals are assumed to pass through a flat fading channel and in the absence of background light. As a consequence, the received signals are only subject to additional shot noises. Thus, in the simulation, the signals are only affected by Poisson distributed shot noise, the nonlinear distortion and clipping distortion (low bias level DCO-OFDM) introduced by SPAD receiver and $T_{ST}$ is assumed to be equal to $T_s$. A PQ SPAD array and an AQ SPAD array are considered with the same parameters as given in Table I.

Fig. 3 shows the average outputs of the PQ SPAD array and the AQ SPAD array as a function of the optical irradiance when $T_s = 1 \text{ ms}$ and $T_s = 1 \mu s$. As noted in Section III B, the nonlinear properties of the SPAD array are shown in Fig. 3.
where the optical irradiance represents the incoming photon rate. Compared with the longer symbol period ($T_s = 1 \text{ ms}$), the maximum photon counts of each symbol, $\mu_{\text{max}}$, are lower when the symbol period is $1 \mu\text{s}$. For the different symbol rates, the nonlinear distortions appear at the same position (-40 dBm). This means that the nonlinearity of PQ and AQ SPAD are predicted to have a significant effect on the performance of the system when the optical irradiance is larger than around -40 dBm.

Fig. 4 and Fig. 5 show the simulation results for bit-error ratio (BER) of the SPAD-based DCO-OFDM and ACO-OFDM systems as a function of the optical irradiance when $T_s = 1 \text{ ms}$ and $T_s = 1 \mu\text{s}$. The performance of 4-QAM DCO-OFDM with 7 dB and 13 dB DC-bias and 4-QAM ACO-OFDM are compared in these figures. In this study, when the optical irradiance is larger than a threshold, BER is below the target BER of $10^{-3}$. This threshold is defined as the minimum power requirement (MPR) of the system.

When the optical irradiance is larger than another threshold, the nonlinear distortion of SPAD receivers occurs, resulting in BER higher than $10^{-3}$. This threshold is defined as the maximum optical irradiance (MOI). The gap between the MPR and the MOI is defined as the low error area (LEA) where the system can maintain a low BER ($< 10^{-3}$). For example, in Fig. 4, after the optical irradiance reaches -85.5 dBm, the BER of 4-QAM DCO-OFDM with 7 dB DC-bias and PQ SPAD is lower than $10^{-3}$, and after the optical irradiance reaches -39.8 dBm, the BER of the same scheme is higher than $10^{-3}$. Thus, the MPR of this scheme is -85.5 dBm; the MOI is -39.8 dBm; and the LEA is 45.7 dB.

From Fig. 4 and Fig. 5, it can be seen that the PQ SPAD-based and the AQ SPAD-based systems have the same BER performances at low optical irradiance (around MPR). This is because in terms of linearity, the PQ SPAD devices have the same performance as the AQ SPAD devices when the optical irradiance is low (Fig. 4). However, as the maximum count rate...
of PQ SPAD is lower than AQ SPAD, the MOIs of PQ-based systems are lower than the AQ-based systems. In addition, since the MPRs of each system are the same, the LEAs of PQ-based systems are also lower than AQ-based systems. As a consequence, the PQ SPAD-based OFDM system is more readily affected by the nonlinear distortion and is more sensitive to optical irradiance.

For different symbol periods ($T_s = 1$ ms and $T_s = 1$ $\mu$s), the nonlinear distortion occurs after the optical irradiance reaches around -40 dBm which matches with the prediction in Fig. 3. However, the schemes with a shorter symbol period ($T_s = 1$ $\mu$s) have higher MPRs than the methods with a longer symbol period ($T_s = 1$ ms). This means that with decreasing $T_s$, LEA reduces, which decreases the range of the received optical power.

Fig. 6 shows the MOI and LEA of the PQ SPAD-based and the AQ-based OFDM systems with different spectral efficiencies and symbol periods ($T_s = 1$ ms and $T_s = 1$ $\mu$s) when the BER is $10^{-3}$. The MOIs of ACO-OFDM are lower than DCO-OFDM. Compared with SPAD-based DCO-OFDM, the ACO-OFDM system encounters the nonlinear distortion earlier since the distortion in ACO-OFDM is doubled (due to the subtraction in demodulation) which increases the probability of bit errors. As ACO-OFDM has a higher power efficiency (lower MPR) than DCO-OFDM, the LEAs of ACO-OFDM become close to the LEAs of DCO-OFDM, as shown in Fig. 6. However, the higher power efficiency of ACO-OFDM has the disadvantage of achieving approximately half of the spectral efficiency of DCO-OFDM. With increasing spectral efficiencies, the MOIs and LEAs of the systems decrease and thus the systems are more susceptible to nonlinear distortions.

In addition, for DCO-OFDM with 7 dB bias, the result for low spectral efficiency (1 and 2 bits/s/Hz) are shown. For short symbol periods ($T_s = 1$ $\mu$s), the spectral efficiencies of the systems are lower than 2 bits/s/Hz. When the spectral efficiencies increase, the MPRs are higher than the MOIs. This means that before BER of the systems reaches $10^{-3}$, the SPAD arrays are almost saturated by the incoming photons and the nonlinear distortion are too dominate to operate the system.

V. CONCLUSION

In this paper, a SPAD-based OFDM system assuming nonlinear distortions introduced by the SPAD device is presented. The nonlinear distortion is caused by the nonlinearity of the passive and active recharged circuit. For the SPAD-based OFDM system, the nonlinear distortion has a significant effect on the BER performance when the optical irradiance is higher than the MOI which is around -40 dBm. Compared with the PQ SPAD-based OFDM system, the system with AQ SPAD has a higher MOI and LEA. This means that AQ SPAD can receive optical signals with higher intensity and has a wider range of useful received optical power. However, these benefits come with the disadvantage of a higher hardware complexity. Compared to DCO-OFDM, SPAD-based ACO-OFDM has lower MPR, MOI and LEA. This means that ACO-OFDM exhibits a higher power efficiency, but is subject to a higher risk of nonlinear distortions affecting the BER performance. In addition, with an increasing symbol rate and spectral efficiency, the SPAD-based OFDM systems are more readily affected by the nonlinear distortion.

The SPAD receiver has a significantly enhanced sensitivity. This means that the SPAD-based OFDM system can be used in long distance transmissions, or it can be used in non-line of sight optical wireless communications, in the uplink when illumination is not essential, or when lights are almost completely dimmed. However, due to such high sensitivity, an appropriate transmission power should be selected carefully so as to avoid the nonlinear distortion.

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